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Introductory and Historical

The problem of this investigation was: To make a careful survey of temperature-discrimination within a range not involving pain or extreme discomfort (16° to 48° Centigrade), with especial reference to the validity of Weber's law; and to note any other matters of importance which might appear in that connection.

The previous work on this aspect of temperature will here be briefly summarized. E. H. Weber[1] himself, in a small

[1] Weber, E. H.: Die Lehre vom Tastsinne und Gemeingefühle (1851); p. 98.

series of trials, dipped the same finger or hand alternately into two jars of water standing side by side. He found that, the whole hand being immersed each time, he could distinguish one-fifth to one-sixth degree, Reaumur, with careful attention. "Most" people, he says, can distinguish with certainty a difference of 2/5° R. He did not find greater differences necessary to distinguish between two temperatures around 14° R. (= 17.5° C.) than in the region of blood-temperature (about 37° C.).

Fechner[2] dipped the first two fingers of the right hand

[2] Fechner, G.T.: Elemente der Psychophysik, (2. Aufl., 1889); Bd I, pp. 201 ff.

into water up to the second joint; alternating between the two

jars until the judgment was formulated. He found that, from 19° to 31° R. (that is, about 24° to 39° C.), the just noticeable differences ("eben merkliche Unterschiede") conformed to Weber's law "sehr wohl", if the magnitude of the stimulus is taken to be the amount by which it exceeds 14.77° R. (the approximate mean between freezing-point and blood-temperature); the Weber ratio for this range he finds to be .03633, which gives a fair agreement between observed and adjusted values, but greatly exceeds the ratio found in this study (.004 to .005). From 10° to 20° R. the discriminable differences were too small to be accurately determined with his thermometer (which was calibrated to 0.5° R. but on which, he says, 0.05° R. could be "sehr wohl geschätzt"). He concludes, in brief, that from 10° R. downward the just noticeable differences increase far more rapidly than is conformable with Weber's law; from 10° to about 19° R. the j.n.d.'s are too minute to be accurately measured with his thermometer; from 19° to 31° R. they agree very well with the law, as noted above; from 31° R. upward, he reports no observations but thinks it "wahrscheinlich" that the j.n.d.'s again increase too rapidly to conform with the law, as they did below 10° R.

Lindemann[1] dipped the right hand, to the wrist, alter-

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[1] Lindemann: De sensu caloris (1857); quoted by Fechner, ibidem; p. 209.

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nately into two jars of water, the one being constant, the other being adjusted to apparent equality with the first (method of

average error). Within the range 26° to 38° C. in one series, and 31° to 43° C. in another, the predominant error was +.05°; above and below these limits the error went as high as .5° to .8°. The number of trials was small, the experimental procedure was apparently very crude, and one may wonder by what means the error was measured to one-hundredth degree C.

Alsberg[1] dipped the index finger, up to the first joint,

[1]Alsberg: Untersuchungen über den Raum- und Temperatursinn (1863); reported by Thunberg in Nagel: Handbuch der Physiologie des Menschen, III. Band; p. 687.

into one glass of water, kept it there for 10 to 20 seconds, then transferred it to a second glass. In this way he distinguished at

10° C.	a difference of	.9°
15°	"	.4°
20°	"	.5°
25°	"	.7°
30°	"	.5°
35°	"	.1°
40°	"	.2°

These results show little regularity, discrimination reaching its maximum at 35°.

Nothnagel's[2] method was the same as Fechner's above, except

[2]Nothnagel: Beiträge zur Physiologie und Pathologie des Temperatur-sinns; Deutsches Archiv für klinische Medizin, II. Band (1867); p. 284.

that only one finger was used instead of two. He found most

accurate discrimination between 27° and 33° C.; slightly less accurate from 33° to 39° and also from 27° to 14°; from 39° to 49° and from 14° to 7° sensitivity decreases rapidly. He does not give the results in detail.

Müller[1] communicates no new observations but concludes,

[1]Müller, G.E.: Zur Grundlegung der Psychophysik (1878); p.

after surveying the discordant results of those previously reported, that the validity of Weber's law in this field is not established (which is indeed obvious).

Other investigations dealing with adaptation, the relative sensitivity of different parts of the body, and the like, need not delay us here. Of all the observations reported on our problem, those of Nothnagel are the most comprehensive and agree most fully with our results.

It is by no means a coincidence that the field of thermal discrimination was but little worked during a period when the facts and methods of psychophysics were being investigated and threshed over in detail. The special difficulties connected with adequate control of the stimuli, with adaptation, and the like, were recognized by Fechner and Müller; temperature thus became, logically, one of the last fields to be surveyed, after methods had been developed and refined in other sense more easily approached. In attacking this problem, the writer hoped, by the use of more delicate methods of temperature regulation than had been employed before, by securing extensive collections of data from the same

observer, by the use of complete series (Vollreihen) in the method of constant stimulus-differences, and by treating his data with the more refined statistical methods of modern psychophysics, to achieve something like a definitive solution of the problem for the range of temperatures, and by the type of method, here employed.

Description of Apparatus

The apparatus in its final form, as used for the method of constant stimulus-differences, by which the central data of this study were secured, consisted of seven jars (capacity, 3-4 gallons each), conveniently arranged on a table. Jars of this capacity were used both to accommodate the necessary heating, regulating and agitating devices and also to secure stability of temperature by means of a relatively large volume of water. The low thermal conductivity of the earthen walls also reduced vacillation in the temperature, and the cylindrical form was an aid to the proper stirring of the water. The tops of the jars were at a convenient level for O (the observer), when standing, to move his hands readily about and immerse them in any desired jar. Each of these was covered with a lid (wood or asbestos); this was provided with a large opening at one side, through which O could immerge his fingers in the water without touching either the lid or the side of the vessel. The surface of the water was two to four cm. from the top of the jar. Extending downward from the lid and supported by it were two heating units, an agitator and a thermostat.

Both of the heating units were ordinary electric bulbs; carbon-filament, as a rule, since this type yields more heat. One of them was a variable heater, that is, it went 'on' and 'off' in rapid alternation, as controlled by the thermostat; the other was 'on' continuously, not being subject to the regulator. This continuous heater had two functions: a) To keep the temperature of

the water always within a few degrees of the point for which it was set. Thus, if the jar was to be held constant at 36.15° C.[1], the room-temperature meantime being 23° to 25°,

[1] Since all temperature values in this study are given in terms of the Centigrade or Celsius scale, the designation "C." will hereafter be omitted.

a bulb was used large enough to hold the water about 9° above the temperature of the air; that is, from 32° to 34°. A small variable heater could then be used for the remaining two or three degrees; the lag[2] of the thermostat and resultant oscil-

[2] Any thermoregulator, of course, has a certain lag, owing to the fact that it does not instantly attain the temperature of the surrounding medium. If this medium warms or cools rapidly, the lag is correspondingly great; the medium will go considerably above or below the 'constant' level before being checked; and thus large oscillations in temperature will result.

lation of the temperature were thus reduced in two complementary ways: the variable being small, the temperature rose slowly when it was 'on'; when the variable was 'off', the fall in temperature was much retarded by the continuous heater; the change being, thus, gradual in each direction, the effect of the lag was correspondingly inconspicuous. As above intimated, the continuous heater-bulb was adjusted to the various temperatures worked with, the smallest procurable (7 1/2 watt Mazda) being used when the water was at or below room-temperature, and 30-60 watt lamps (carbon filament) at higher levels. The second function of the continuous heater was b) To keep the interior of the jar light at all times. In this way O could properly guide his hand in the process of immersion--a completely dark jar was found to be inconvenient in

that respect. This illumination was not wholly constant, being slightly greater when the variable lamp was also 'on'. No reason appeared to consider this small difference in illumination as affecting O's judgment. It might be thought that, since the temperature rose as long as the jar was lighter, O's estimate would be subtly affected by a knowledge of this fact. As to that, the rapid alternation of 'on' and 'off'--many times per minute, as a rule--served, in the opinion of the principal O, effectually to baffle any such influence. - The variable lamp, it may be said, ranged from 30 to 100 watts (carbon filament).

The agitator or stirrer was of a simple screw-propeller type, having three blades so adjusted as to give constant movement of the water, but without causing superficial waves or strong currents, which were found by O to be disturbing. It is obvious that more heat will be exchanged between water and hand if the water streams rapidly over the fingers than if it moves slowly. It was, therefore, necessary to have the stirring as nearly equal in all jars as possible. No attempt was made accurately to determine and equate the amount of water handled by the stirrers in unit time; but to secure approximately equal movement movement in each jar, 1) the stirrers were of the same size; 2) the curvature of the flanges was adjusted as accurately as could be done without special instruments; 3) they had the same position in each jar, the flanges being about three cm. above the mid-point of the bottom; and 4) they were all driven at the same rate. Frequent observation with both eye and hand

revealed no apparent, or at least gross, differences in the rate of flow of the water. That the stirring fully served its function of keeping the temperature of the water uniform throughout the vessel was shown in repeated tests with the Beckmann;[1] which revealed differences in no case exceeding

[1] A "Beckmann" is a mercury-thermometer, calibrated in hundredths of a degree, and much used for measuring small changes in temperature.

.005° between the upper three cm. of water and that at the bottom of the jar. The stirrers were turned by means of belts passing over a steel shaft fixed on the rear of the table and run by a motor.

The thermostatic device was similar in type to the De Khotinsky mercury regulator,[2] but was designed, after numerous

[2] Designed by A. de Khotinsky; manufactured and sold by Central Scientific Co., Chicago.

preliminary trials and modifications, specially for this work.[3]

[3] The writer wishes to acknowledge the assistance of Dr. Marion Hollingworth, Department of Chemistry, Ohio State University, in solving certain technical difficulties connected with making these instruments.

The form finally adopted consists of two upright tubes, supported by a central stalk. This was securely sealed at its top to a circular metal piece, which in turn was firmly fastened to the supporting lid. The vertical tubes were filled with toluene--a substance often used by physical chemists for this purpose.[4]

[4] A filler for thermostats should have several properties:

1) Large coefficient of expansion;
2) Small heat capacity, so that a slight influx of heat-energy will produce a relatively large increase in temperature;
3) Slight compressibility;
4) High thermal conductivity.

No substance has all these properties in ideal measure; but toluene and related liquids, such as chloroform, combine them to relatively high degrees. Cf. Ostwald-Luther: Physiko-chemische Messungen, 3. Aufl.; p. 113.

The stalk and the base of the tubes were filled with mercury which, by the expansion of the toluene, was forced up into a glass capillary above the lid (diameter of capillary tube, about one mm.). The capillary was fitted with a threaded cap which screwed into the metal head-piece, and could thus be adjusted at will. A piece of German-silver wire, bent round so as to keep the contact-point near the center of the capillary, extended down into it and made contact with the mercury meniscus. This contact, by closing an electric circuit, operated an electro-magnetic relay, which cut off the variable heater; as soon as the mercury sank below contact, the lamp again came on. The smallest practicable amount of current was used in order to reduce the break-spark, and consequent oxidation of mercury at the contact-point, to a minimum. Three to four thousand ohms resistance (small Mazda lamps) were inserted into each circuit from thermostat to relay, so that the amount of current used, by several tests with the ammeter, was .03 to .05 ampere; the energy used to operate the relay was, thus, .2 to .4 watt. By repeated attention, the mercury-contact was kept clean and dry, so that, as a rule, the break was easy and the spark light. To eliminate "sticking", which delays the break at the point of

contact, the natural vibration of the regulators, caused by the running of the apparatus, was accentuated by placing the table-legs on small rubber pads; the resultant "tremor" of the table reduced clinging by the mercury so that, as a rule, the contact made and broke some ten to twenty times a minute throughout the working period. The incessant snapping of the relays, in fact, closely resembled the clicking of a battery of telegraph keys. With the regulators working in this way, as a number of tests at different times and temperatures showed, the fluctuations were less than $\pm$ .005°; even though the temperature of the jars might rise or fall slightly within an afternoon, the ordinary oscillation, in favorable conditions, did not exceed the above figure.[1] Favorable conditions were

[1]The reader will clearly distinguish the regular, make-and-break oscillations, due to lag in the thermostat, from the gradual rise or fall over a period of some hours, due to various causes presented below (p. 13 f.); the two are independent phenomena. To illustrate the distinction: at 13 M. a regulator may oscillate .005° above and below 28.10°; at 5 pm the temperature may have risen to 28.13°, about which point it will still oscillate.005°.

not, of course, always present; as anyone familiar with such work knows, relatively sensitive instruments, especially when made and operated in a situation by no means ideal, are correspondingly subject to slight defects or difficulties. For some parts of an afternoon or on particular days, certain thermostats showed more lag than usual; sometimes the cause could not be at once detected; or a series could not be stopped to remedy the difficulty; so that, a few times, the fluctuation reached as

such as .04° or .05° on each side of the stated value. It must be understood, however, that this degree of latitude was distinctly rare.[1] Resorting to estimate,[2] the writer

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[1] On this point the writer has no systematic data to represent. To have determined accurately the average oscillation of the regulators would have demanded rather extensive measurements for each jar at each temperature--an additional burden which, in the circumstances, did not seem practicable.

[2] The writer needs no reminder that estimates are precarious; his statement above, however, is far from a mere guess. It rests upon:

a) His own observations with the Beckmann, above alluded to, of the actual deviations both when the regulators were working precisely (with rapid sequence of 'on' and 'off') and also when their 'rhythm' was slower; in this way he learned to appraise, with some accuracy, the range in temperature from the rate of alternation of the variable lamp. By means of this direct index, therefore, the latitude of change, could be, and was, followed every hour of every working-period; any slowing of the normal 'rhythm' was at once apparent; the cause could be sought and, if possible, remedied, either between series or, more throughly, at the close of the day.

b) His repeated inspection of the mercury contacts-one or more times per hour during the working-period. This constant attention also helped to reveal any failure of the regulators at their critical points.

Every effort was thus made to keep the apparatus always at maximal efficiency; and it is primarily upon this personal scrutiny of the instruments every hour of every day of the period covered by these observations, that the writer's estimates above are based.

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considers the statement conservative that the maximal oscillation in 50% of the cases used in the data herewith presented did not exceed .005°, and in 90% of the cases, did not exceed .025°, on each side of the stated value; so that a temperature marked at 36.05° would, in about half the cases, range from 36.045° to 36.055°, and in only a small fraction of cases would range beyond 36.025° and 36.075°.

It was the custom to read and record the temperatures before and after the experiments of each day. In the later stages, additional checks were often taken during the working-period, but an initial and final reading were invariably secured. In many cases the final reading showed a rise or fall of several hundredths, sometimes more, apart from the oscillations above discussed. When this change from one reading to another exceeded certain carefully-defined limits, the values were not used (see p. 150, below). Among the observed causes of a decrease between readings were: 1) Lag in the original setting. In most cases the working temperature was above that of the room, so that each morning the water had to be raised to the desired level. If the regulator was at once set, the level it actually held was usually found to be a few points lower. The water having been quickly heated up, the toluene was still slightly cooler; the thermostat was thus really set to this cooler level. Ordinarily the regulators were readjusted until they had attained stability at the desired point; but in a few cases, because of limited time or negligence of the assistant, this factor accounted for a difference of several hundredths between the original and a later reading. It may be noted that, in the later stages of the work (Group II Table 1), a second reading by another assistant was regularly[1] secured

[1] That is, in 25 cases out of 30 - 83⅓.

before, or at the time when, experiments began; so that this factor was not then present. 2) The repeated spark at the contact--as above noted, often exceeding ten to twenty times per minute for several hours--inevitably oxidized some of the

mercury; this ends sometimes collected on the inside of the capillary around the contact; being a conductor, the deposit might close the circuit between the wire and mercury before they came in actual contact, and thus lower the temperature for points. 3) Several times, apparently, a small particle of mercury, accidentally lodged in the place above the capillary, dropped down to the contact, and thus lowered the level for an three hundredths.

The causes of an increase between readings were such as: 1) Lag in original setting, as above. If the regulator was set down from a higher level, the lag would operate as above described, but in the opposite direction. 2) The mercury-oxidation usually brought about a slow rise during the day. This increase may be taken as uniform over the whole interval, since the rate of oxidation was approximately constant. This factor, in the writer's judgment, was by far the commonest cause of increase between readings, but its influence probably never exceeded 0.02° to 0.04° in an afternoon. 3) Several times, a slight leakage of mercury, usually through the seal between glass tube and metal head-piece, caused a small rise.

Since the commonest and most important of these factors, in the writer's judgment, (No. 2, in each case), was reasonably constant in its effect, and therefore justified linear interpolation between the two end-values of the interval; and since no values deviating more than 0.10° between consecutive checks appear in the data actually used, the writer believes that these factors introduce no appreciable error into the results.

sions.

In this connection the accompanying Table,(Table I), showing the average rise or fall per hour for the whole period covered by these data, should be consulted. It will be seen that the total distribution for each group, especially for the first, is markedly of the $-Cum, corresponding to Pearson's type III; (from inspection; there was no occasion to fit the curves. Cf. Elderton: Frequency-Curves and Correlation; p. 65). The medians are, therefore, markedly smaller than the means.

It will also be noted that the values for the second phase of the experiment - 48° to 28° - are about twice as large as for Group I. Thus, the "combined" mean and median for Group I are 0.57 and 0.33, for Group II, 1.18 and 0.86, respectively. Do these larger deviations in Group II indicate a decrease in the constancy of the thermostats during the second phase?

Two causes appear to the writer to contribute to this increase in the average deviation: 1) The first and, in the writer's judgment, more important of these shows the above difference to be, in large measure, an artifact. In the first group only two readings a day were secured - before and after the working-period (afternoon); in the second, additional checks were often taken during the afternoon, between series. (Exact figures: in Group I, additional readings were taken on 2 days out of 51 - 4 %; in Group II, on 17 days out of 30 - 57 %; a ratio of 14 to 1). The purpose of these extra readings, of course, was to follow more closely any possible changes in

Notes to Table I: The table following is presented for the purpose of showing the accuracy of the thermometer (that is, their average amount of change, positive or negative, see below) for the whole period in which the 'constant-stimulus-difference' method was used.

The reader will note that these changes refer to the general rise or fall over a period of time, not to momentary oscillations. The first column contains the temperature-levels, in the order actually used; the "decrease" columns present all cases in which a reading showed a decrease from the one next preceding; the "increase" column, similarly, all cases showing an increase; the "no change", all cases where two successive readings were identical; the "combined values", finally, include these three groups. The values are to be read in hundredths of one-twentieth degree. Thus, in the first row, after 30°, it appears that, in 33 cases showing a decrease, the mean and modal decrease were 0.77 and 0.55, respectively, of one-twentieth degree. The "General Average" gives the (weighted) means and general medians for all the cases in that group. In the section, "Distribution for Combined Values", on page 17, the column-heads give the deviation, again, in hundredths of one-twentieth degree; the entries show the number of cases.

As is to be expected, there were a few cases in which, for one reason or other, the discrepancy between two successive readings was so large that the values could not safely be used. It became necessary, therefore, in tabulating the data, to fix the exact limits of acceptable change; for a full statement of these criteria of acceptance, see page 150. The same criteria were used in forming this table; so that extreme changes, whose values were omitted in determining the means, are also excluded here. The table, therefore, is intended to give an accurate summary of all the deviations that actually appeared between successive readings, under the same conditions and subject to the same criteria, as apply to the data herein presented.

TABLE I

Table Showing Amounts of Change per Hour in Thermostats

(Constant Stimulus Differences)

All Values Given in Hundredths of One-Hundredth Degree

Group I

Temperature Level	Decrease (Minus) H	Decrease (Minus) Md	Decrease (Minus) No. Cases	Increase (Plus) H	Increase (Plus) Md	Increase (Plus) No. Cases	No Change No. Cases	Combined Values (Minus, Plus, Zero) H	Combined Values (Minus, Plus, Zero) Md	Combined Values (Minus, Plus, Zero) No. Cases
32°	0.77	0.33	33	0.70	0.40	91	15	0.64	0.33	139
28°	0.49	0.50	74	0.84	0.67	96	45	0.54	0.33	215
24°	0.64	0.33	52	0.79	0.50	86	46	0.55	0.33	184
20°	0.62	0.50	84	1.01	0.67	168	34	0.78	0.57	286
16°	0.38	0.17	29	0.78	0.67	83	24	0.56	0.33	136
20°	0.45	0.40	71	0.56	0.33	114	64	0.39	0.33	249
24°	0.54	0.40	39	0.70	0.54	54	28	0.49	0.25	121
28°	0.54	0.60	13	0.84	0.80	38	18	0.57	0.50	69
General Averages:	0.55	0.40	395	0.80	0.60	730	274	0.57	0.33	1399

Group II

40°	0.79	0.50	14	1.19	0.76	36	6	0.96	0.40	56
44°	0.41	0.43	39	1.42	1.25	38	8	0.91	0.67	77
48°	1.34	1.00	32	1.65	1.00	24	4	1.38	1.00	60
36°	1.78	1.00	32	1.12	1.00	25	10	1.27	1.00	67
32°	1.13	0.50	23	1.31	1.00	16	5	1.07	1.00	44
28°	2.40	2.33	15	1.46	1.00	24	5	1.61	1.33	44
General Averages:	1.22	0.67	155	1.35	1.00	163	30	1.18	0.86	348

TABLE I (Continued)

Group I

Temperature Level	Distributions for Combined Values Column Heads give Change in 0.01° Entries give Number of Cases							
	0.00 to 0.49	0.50 to 0.99	1.00 to 1.49	1.50 to 1.99	2.00 to 2.49	2.50 to 2.99	3.00 to 3.49	Over 3.50
32°	79	19	27	11	0	3	0	0
28°	114	66	16	6	3	5	5	0
24°	116	30	9	20	6	3	0	0
20°	121	75	43	26	6	6	7	2
16°	72	24	33	0	5	0	0	0
20°	103	40	22	0	0	4	0	0
24°	75	32	4	7	3	0	0	0
28°	31	20	13	5	0	0	0	0
General Averages:	791	306	169	75	23	21	12	2

Group II

48°	29	4	10	0	9	3	0	1
44°	29	24	5	10	3	3	1	2
40°	9	15	17	0	11	0	5	3
36°	19	10	18	3	6	5	1	5
32°	14	12	8	5	0	0	3	2
28°	9	7	12	5	4	3	4	4
General Averages:	105	72	70	23	33	14	14	17

temperature, but they also served to increase artificially the apparent deviation per hour. To illustrate: Given a jar oscillating between 40.10° and 40.11°; at 1 pm it reads 40.10°, at 5 pm it reads 40.11°; mean change per hour = .05 hundredth. If two extra readings are interpolated, we are likely to get a series like: 40.10°, 40.11°, 40.10°, 40.11°; mean change per hour = .75 hundredth - just three times as much as before. Thus, by ordinary probability, it is evident that, if a jar ranges between 40.10° and 40.11° and the temperature is read to the nearest hundredth, it will be 40.10° in about half of all cases and 40.11° in the other half. Hence, the more readings in a given space of time, the greater will be the apparent change per hour - an effect due merely to oscillation and not to actual change of level. 2) In addition, it seemed to be necessary hereto to keep the regulators at optimal efficiency during the second phase - when temperatures - than earlier. The writer has no figures to prove this difference nor its extent; but he gained the impression, at the time, that the readings required more attention and adjustment then earlier. The high temperatures, for example, once or twice caused trouble with the seal between glass and hand-piece in the thermostats. One of the assistants at this stage was also less competent than his predecessor had been - a fact which involved a greater burden on the writer. How much this may have influenced his impression can not be determined.

There seems to consider another point in this connec-

tion. When tabulating the data from a series of judgments involving a given jar, in case its temperature had not been read at the close of that particular series, the writer regularly derived the value by linear interpolation between the readings next before and after. The course of change between the two readings was thus assumed to be uniform; but since this cannot be proved - however plausible it may be in any given case - it is of some consequence to know the average change per average interval; in order that we may fix the range by which the interpolated value is likely to differ from the 'true' temperature at that time. To that end, the following table is presented, which gives the time in hours between successive temperature-checks for both groups I and II.

TABLE II

Intervals between Successive Temperature-Checks

Length in Hours	7	6	5	4	3	2	1	No. Cases	Mean Interval	Median Interval
Group I	1	29	13	7	3	2	2	57	5.1	5.6
Group II	1	1	5	3	12	6	11	41	2.8	2.6

If we now calculate the change per interval for each group, we get, in hundredths of a degree:

	I	II
Mean change per mean interval	2.9	3.3
Median change per median interval	1.8	2.2

(The value, 2.9, is obtained by multiplying the mean change per hour, 0.57, by the mean length in hours, 5.1; likewise with the others.)

It then appears that the average change between readings, for both groups, is about 0.02° to 0.03°. The strikingly greater change per hour in group II is here neutralized by the more frequent readings and consequent shorter intervals; so that for our ultimate purpose of getting, as accurately as possible, the true temperature for each series, the two are, but for a negligible difference, equally good. If we now assume that, when a jar reads, say, 32.10° at 1 pm and 32.15° at 3 pm, it has not appreciably[1] changed outside those limits during the interval,* it follows that the mean maximal discrepancy between the interpolated and the 'true' values during that time will be just one-half the total change that has occurred.[3] In the case of the four averages above, it runs

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[1]That is, beyond the amount of its regular oscillation.

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*The only possible exception lay in such as the thermostats were regularly subject to control and repeated resetting every working-period, as already adverted to above.

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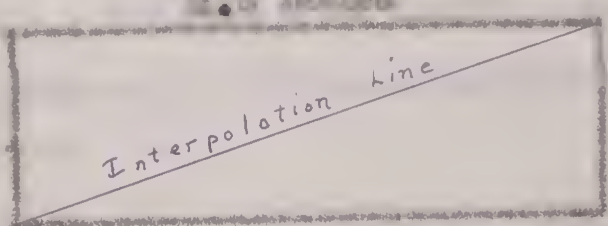
[3]This will be clear from the adjoining figure.

2.5 hours



0.033°

If a reading shows an increase of 0.033° over the one taken 2.5 hours earlier, the actual temperature has, by our assumption, not transcended these extremes during that time. It is obvious that the mean distance from interpolation-line to upper and lower limits of range is one-half the total range 0.033°.

from 0.009° to 0.016°; in other words, the maximal error due to interpolation is, on the average, about 1 to 1.5 hundredths. In view of these facts, therefore, the writer believes that his policy of fixing temperature values by interpolation, when necessary, was entirely justified.

The writer holds, in sum, that, despite reported difficulties in achieving and maintaining the desired degree of precision, the accuracy of the regulators, while by no means the finest attainable, was at least adequate to the demands of this problem. One cannot, certainly, without a physical equipment quite beyond the resources of the ordinary Psychological laboratory, hope to attain a degree of stability in temperature-constants such as can be got with relative ease, for example, in metal weights; but at least the variation of the constants

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ref. Urban, F.M.: The Application of Statistical Methods of Psychophysics, (1908); pp. 1, ff. and 173.

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should be only a fraction of 0°.1 limen, if reliable results are to be possible. Inasmuch as preliminary trials revealed a limen considerably less than 0.10° under optimal conditions, it is evident that a variation of 0.10° - the limit of accuracy in ordinary commercial thermo-regulators - is quite too large.

Owing to laboratory limitations, the cooling device was primitive and cannot be called satisfactory from the viewpoint of general efficiency or convenience; but, by dint of careful control, it was made to give reliable service and, from all

data at hand, the variations of temperature when it was used did not exceed the average limits for group I. (Exact figures: change per hour for 16° and 20° - the two levels at which the cooling device was principally used - was: Mean = 0.59, Median = 0.40; as compared with 0.57 and 0.33 for the whole of group I and with 0.56 and 0.33 for group I, excluding 16° and 20°. Cf. Table I, p. 17). It consisted of a large reservoir, filled with ice and water, which was placed above and behind the table; the cold water from it was siphoned into each jar through individual rubber tubes, the flow being regulated by screw-clamps. At the setting of the thermostats of a morning the flow was adjusted until each jar remained constant at the desired level. Persistent watchfulness was employed to make sure that the mercury in each regulator stayed exactly at contact with the wire. If, in any case, it fell below contact (through too great inflow of cold water) or rose above it (through deficient inflow), the jar was not used until contact had been restored. The surplus water was siphoned out into a receptacle below. This outflow was also controlled by screw-clamps, in such way as to maintain the water at about the same level in all jars. For wholly satisfactory work of this sort, a refrigerating device should be available, much more delicately regulable than flowing water; but the evidence above cited tends to prove that the reliability of the data was in no wise affected by the limitations of the cooling apparatus.

For purposes of accurate measurement, a Beckman ther-

mometer, as above noted, was indispensable; one was secured, by loan, for the whole period of the experiment, through the kindness of the chemistry department of Ohio State University. In the method of limits, where the temperature of the variable rises or falls continuously within a given series, the Beckmann was useful for getting the temperature quickly and accurately at the instant of O's judgment. It was of the highest value in the 'constant stimulus-differences' method as well, for convenient and accurate setting and checking of the constants.

A light was suspended about three feet above the table, shaded from O's eyes but illuminating the whole apparatus. In the later stages (group II) a thermometer was placed above the table from which the air-temperature was daily recorded.

Description of Procedure in Collecting Data

The general procedure, whose definitive form was reached in the method of constant stimulus-differences, will here be exhibited in detail; any departure therefrom, in the use of other methods, will be noted in connection with the appropriate data.

In using this method, by which the great bulk of the data was secured upon which the conclusions of this study rest, the writer[1] did practically all the observing himself, as will appear from the table of results. The data by the method of limits are largely from other observers, but include some from D-O. For the director of an experiment to do most of the observing, especially when the procedure must be "without knowledge", is not an ideal method. It was adopted in this study, not from principle, but of necessity. Psychophysical investigations being somewhat tedious, and time-consuming, for O, to a degree unknown in ordinary psychological experimentation; being also out of line with the interests of most present-day students of Psychology in this country, it was found impracticable to get O's who could give the time and devotion that such quantitative work inexorably demands. O must be prepared to make his service, as Fechner relates[2] in his own case,

[1] Who will hereafter be distinguished from other O's by the symbol D-O, in virtue of his double position as director of the experiment, and observer.

[2] Fechner, G.T.: Elemente der Psychophysik, (2 Aufl., 1889) I, p. 92.

"[illegible]" for prolonged periods. As a result, the writer decided to make his own observations as complete and extensive as possible, in lieu of fragmentary results from a number of other O's. His data, accumulated in a period of over four months, comprise, for the whole of that time, from 17 to 25 hours per week, that is, three to four hours per day, of actual observation.

The merits and demerits of this policy claim a few words. In favor of E's observing are several undoubted benefits: 1) Extensive series of data can thus be secured from the same subject — a cardinal necessity in this type of problem. The irregular and limited cooperations often resulting when the convenience of other O's must constantly be consulted are largely avoided. 2) E's interest in the process of taking data is supported by incentives which no other O can be expected to have in the same degree; he is therefore likely to be more attentive, accurate and consistent in his judgments; his wider knowledge of the topic enables him better to note and interpret the auxiliary factors in the problem under examination. 3) E thus receives, finally, a concrete and detailed knowledge of the whole process upon which his data depend, such as he is not likely to get if he confines himself to recording the observations and comments of others. The policy, on the other hand, has no slight disadvantages: 1) The data being supplied largely by one O, critical evaluation of them with data drawn from other sources under like conditions is impossible. Generalization becomes more precarious, and the insight to be gained from comments by other

technical observers is lacking. This shortcoming does not, however, appear to be critical here, since our primal concern is with the relative performance of the same O under different conditions, rather than that of different O's under the same conditions. The writer's absolute thermal sensitivity may well differ from that of other O's; but our first interest, in this study, lies elsewhere. 2) The need of keeping E-O in ignorance of the actual temperature-differences made it necessary to commit the setting of the jars to assistants. These were selected from students and graduates in the department,[1] and properly remunerated. Their service was, in nearly

[1]of Psychology, Ohio State University, where the experimentation was conducted.

all cases, quite commensurate with their tasks;[2] but for E

[2]Those engaged in the vital work of setting and checking the temperatures were chosen with especial care and consisted of: Miss Ruth Miller, Laboratory Assistant in Psychology; Miss Edna Willis, school teacher and graduate student; Mr. [illegible] Wagner, then graduate student; Miss Doyo, school teacher and summer session student; all the above were mature and capable persons. Further, R.L. Gordon and C.O. Schwank, undergraduates, whose care and fidelity were most exceptional; H., undergraduate of ordinary calibre, considerably less good than the two just mentioned; and two others, N. and G., whose work in this respect - very limited in amount, moreover - was, so far as the writer could judge, quite satisfactory.

In group I, the jars were regularly set by either Gordon or Schwank and checked at the close of the afternoon by one of them or (usually) by Miller; in group II, the jars were set by H. but regularly checked (in all but [illegible] cases out of 30) before, or at the beginning of, the working-period by Wagner (usually), Willis, N. or G.; the final check was by Doyo (usually) or Miller. It will thus be seen that, in the case of the only assistant who was less satisfactory (H.), an additional check, then or before observing began, was taken in 9[illegible] % of the cases. Further, the full discussion above of the mean changes between readings may lie in evidence that the assistants did their work very well indeed. To all those named above, the writer wishes here, also, to acknowledge his indebtedness; in especial

degree to Miss Miller, whose unfailing interest and help were invaluable.

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to have personal charge of such matters is, in general, to be preferred. The actual procedure will be set forth in the second paragraph below. 3) A third weakness of this method was, that the results could not be tabulated and treated until experimentation was completed. This much delayed the final phases of the work and compelled [illegible] to proceed somewhat 'in the dark' without the guidance which early data can yield for the future course of an investigation.[1] In spite of these

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[1] This sentence must not be interpreted to mean that the writer was here entering upon his experimental work with a kind of self-imposed blindness. A large body of material had already been collected from four different [illegible]'s by the method of limits, during which period the writer had personally attended to every detail from apparatus to protocols; and had also spent about three to four weeks in personal observing by the same method. He thus had abundant opportunity to acquaint himself with almost every problem of apparatus or procedure, before ever beginning with the 'constant stimulus-difference' method.

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facts, this policy was consistently held, to prevent any disturbance of future results by the knowledge of previous ones. Thus, 1) it may be urged, in a study which seeks to determine a law, already formulated and much threshed over for other senses, holds also in this field, that knowledge by [illegible] of preceding results might subtly warp his future observations in favor of the theoretical demands of the law. It may be granted that such an effect is not inconceivable; but the complexity of the situation was so great as to make it, in the writer's opinion, wholly negligible. 2) Much more important was this policy in respect of the constant errors due to method. Defi-

ate knowledge of the presence and extent of such an error leads almost certainly to one of two results; either to its gradual elimination as O learns to make allowance for it in his judgments, or to its accentuation through O's effort to avoid the other alternative.

In sum, therefore, the writer submits that none of the 'a priori hypotheses' just cited were crucial here: the first, for the reason given above; the second, owing to the competence and fidelity of the assistants; the third, because of the writer's extensive preliminary experience with the whole range of problem and procedure involved.

The following temperatures were used in the order shown:

Group I (February 10 – June 8, 1919)[1] 32*, 28*, 34*, 30, 16, 20, 24, 26.

Group II (July 2 – August 11, 1919) 40, 44, 50, 36, 52, 21.

[1]The data from the starred values are used only for comparative purposes, owing to the predominance of practice effects.

The method (whether simultaneous or successive presentation of the stimuli, as well as the adaptation temperature) and, more especially, the constants to be used each day were fixed in advance by E-O in accord with a systematic schedule; the assistant then arranged these temperatures in an order known only to himself among the several jars; when comparing two stimuli, therefore, E-O had no clue whatever to their actual disparity.[1] E-O thus knew the original temperatures

[1]Several times, by inadvertence, the actual difference between two jars became known to E-O.

that had been set for a series and about what differences were present; but he took every precaution not to know how these intervals were distributed. E-0 thus had no means of knowing whether the separation of two given jars was, say, ± 0.05 or ± 0.15 or that; his information was strictly limited to these points: 1) Roughly, how many intervals of each size - 0.05°, 0.10°, and so on - were represented in the series; and, more specifically, 2) whether any two jars had been set equal, and 3) that no two jars differed, in the original setting, by more than a certain amount, say, 0.25°.

The question then arises: Given this order of knowledge by E-0, was the procedure still 'instructive' in every essential part? Considering point 2) above we may ask specifically: Did E-0's knowledge that some or fewer 0.00°-intervals were scheduled for a given series affect in any way the probability of a E-judgment[1] in that series? E-0 might, for ex-

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[1]The 'Equal'-judgment is not specially considered here for the reason that, by the aims of the practice period, 'Equal' had been almost wholly supplanted by 'Doubtful'. For example, the records were examined in two places chosen at random to determine the relative number of D- and E-judgments; the figures are:

Group I (first ten series in May) D = 119 E = 0.
Group II (last ten series in July) D = 300 E = 0.

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ample, tend to give more D-judgments than he knew that several pairs of jars had been set equal than if only one pair were the case; or he might tend to compensate and thereby displace his judgments in the opposite way.

On this point the writer can offer almost no data because regularly two jars, at least, were not equal; so that

in only two cases can he submit, for comparison, series which 1) are free from practice effects, 2) use the same method at the same temperature, and 3) have differing percentages of 0.00°-intervals; see Table III. This table shows, that an increase in the proportion of 0.00°-intervals (Column 1, 0.05 to 0.10; Column 3, 0.15 to 0.21, 0.26 to 0.33) is attended by a decrease in the percent of D-judgments (0.41 to 0.35, 0.45 to 0.29, 0.36 to 0.31) in three cases; by an increase in one case (Columns 5 and 6, 0.19 to 0.34 and 0.25 to 0.26); and the same proportion by an increase in one case (Columns 1 and 2, 0.13 to 0.13 and 0.26 to 0.34). It is obvious that nothing like a uniform relation appears in this table; and while the probative force of these figures is by no means high, what evidential value they have tends to negate the existence of any correlation between these two factors.

Some other facts bearing on this general question are, however, more conclusive. 1) As has already been elaborated, the temperatures often rose or fell between successive checks by one or more hundredths; the intervals between them were thereby correspondingly altered. Thus, of two jars differing originally by 0.05°, if one rises 0.03° and the other drops 0.01° the interval is reduced to 0.01° (counted as 0.00° in tabulating). Inasmuch as these slight changes were almost always present in one or more jars, the original distribution of intervals seldom persisted unchanged throughout a working-period; neither the number of 0.00°-, nor of any other, intervals could be considered perfectly fixed; hence D-0, being

TABLE III

	0.00°		0.05°		0.10°	
	Percent .00°-differences scheduled	Percent D among .00°-comparisons	Ratio .00°- to .05°-intervals	Percent D among .05°-comparisons	Ratio .00°- to .10°-intervals	Percent D among .10°-comparisons
	1	2	3	4	5	6
20°	0.05	0.41	0.15	0.45	0.19	0.25
20°	0.10	0.35	0.21	0.29	0.34	0.26
25°	0.13	0.26	0.26	0.34		
25°	0.13	0.34	0.33	0.31		

Notes: This table bears on the question: Did the fact that S-O knew about how many 0.00°-differences were scheduled for a given series affect the frequency of his D-judgments in that series? At the left appear the temperatures - 20° and 25° - for which two complete sets of data, taken separately and with unlike percentages of 0.00°-differences, are available. Only the figures for the three intervals - 0.00°, 0.05° and 0.10° - are shown; column 1 gives the percentage of 0.00°-intervals booked for a given set and column 2 the percentage of D-judgments from all 0.00°-comparisons in that set; 3 shows the number of 0.00°-intervals divided by the number of 0.05°-intervals in the schedule, and 4 how often a D-judgment followed from all 0.05°-comparisons; 5 and 6, finally give the same facts about 0.10°. Thus, the first row was derived as follows: 75 intervals were scheduled for the first series at 20°; of these 4 (= 0.05) had a difference of 0.00°, 25 (= 0.33) of 0.05°, 20 (= 0.27) of 0.10°. In column 1 appears the 0.05°, in column 3, 0.15 (equals 0.05 divided by 0.33), in column 5, 0.19 (equals 0.05 over 0.27). The judgment D was given to 0.41 of all 0.00°-comparisons (column 2), to 0.45 of all 0.05°-comparisons (column 4), and to 0.25 of all 0.10°-comparisons (column 6).

No figures are given for 0.10° opposite 25°, because not every one of the series forming these two sets had a +0.10° and -0.10° value represented.

fully aware of these facts, was in no way prevented from allowing his knowledge of the original setting to complicate and guide his judgments. 2) Because of constant errors due to method, the modal I-judgment often fell above or below 0.00°. The accompanying table, Table IV, shows how the crude scores are distributed for the various temperature-levels and time-and-order-orders in our data. It will be noted that only 12 of 50 (36 %) fell at 0.00°; that is, in 64 % of cases, a I-judgment was less likely to appear between temperatures differing by 0.00° than between those separated by 0.05° or more. Nor, even if I-O knew that one series had twice as many 0.00°-intervals as another, he was also thoroughly aware that this fact had no definite causal relation with the number of I-judgments at all; it might increase or decrease their number, according to the extent and direction of the constant errors. Since, further, the amount of this constant tendency was wholly unknown to him, while its direction could only be guessed at, I-O was well warranted, in the writer's opinion, almost any tendency to stay, artificially, his process of judging on the basis of the number of 0.00°-, or of any other, intervals. In brief, the writer is convinced, both in theory and from his experience as I-O, that the operation of these constant tendencies, whose degree and direction were unknown to him, served effectually to baffle any evasion or unconscious tendency to guide his judging by the light of what knowledge he had about the original distribution of the intervals. 3) A number of times, especially during work in Group II, one or more

TABLE IV

Distribution of Group Modes for I-Judgments

+0.25°	+0.15°	+0.10°	+0.05°	0.00°	−0.05°	−0.10°	−0.15°	−0.20°
1	1	6	7	11	10	3	1	2

Note: This table gives the distribution of the group modes for the four combinations of time- and space-errors at all of the temperatures used, including practice series - 50 cases in all. When the frequency curve was slightly bi-modal, the mode with larger frequency is here recorded, while if both modes were equal the one based on the larger number of cases is given; one case of bimodality, finally - more precisely of I-judgments based on the same number of cases - is omitted.

We read the table: The figure 1 under +0.20° indicates that, in one case, the largest percentage of I-judgments appeared when the comparison jar was 0.20° above the standard; the 11 under 0.00° means that, in 11 cases, the highest frequency of I-judgments resulted when the comparison jar was equal to the standard; and so on.

The two time-orders - standard preceding and following; expert on temperature - and the two space-orders - standard to right and to left of comparison jar - generally yield different modes, owing to the presence of constant tendencies, to be discussed later (page 120); e.g., the disparity between the modes for the two time-orders is largely due to the exposure of the judging-surface to the air during the process of comparison; the effect of this exposure is equal and opposite in the two orders. The interval between adaptation- and air-temperature not being the same at all levels, however, the amount of this effect varies also.

thermostats were reset, and checked by assistants, between series in the course of the working-period; in this way a new distribution of intervals was secured without any knowledge by E-O as to its extent. 4) An inspection of Table V will reveal that the percentages of 0.00°, 0.05°, 0.10°, and larger, intervals, as scheduled and known to E-O, did not greatly vary from one temperature level to another, if the practice series are disregarded. More small differences were used at the levels where discrimination became more accurate; but apart from this it will be noted that the distribution of intervals changes but little, and slowly, from one level to another. This similarity in the scheduled distributions served also to reduce any possible effect that the knowledge of them might have. 5) Up to this point the writer has tried to be more than fair to any possible objections that might be raised against his method, and to array enough facts to disprove, in his opinion, any influence by E-O's knowledge upon his judgments, even and this knowledge been at all times clearly before his mind; but in fact nothing of this sort was true. To assume that E-O clearly knew or recurrently called to mind the distribution of intervals would be totally erroneous. Not only did the actual distribution of differences from day to day, as indicated under 4) above, vary little (in fact, it remained practically identical at any given temperature level), but it also rarely came to mind at all. The writer wishes to emphasize that he practically never thought of the differences, apart from the way they appeared in the actual process of judging. By the close of the practice period, the judging process

TABLE 7

Distribution of Intervals from Original Schedule

(In Percentages)

Size of Intervals

Temperatures	Practice Series 0.00°	0.05°	0.10°	0.15°	0.20°	0.25°	Over 0.25°	Number of Intervals
32°	0.03	0.13	0.13	0.17	0.13	0.13	0.27	30
28°	0.07	0.27	0.20	0.13	0.13	0.13	0.07	30
24°	0.00	0.23	0.25	0.20	0.15	0.08	0.05	40
				Group I				
20°	0.03	0.33	0.27	0.20	0.13	0.01	0.03	75
16°	0.03	0.40	0.33	0.13	0.07	–	–	40
20°	0.10	0.46	0.25	0.13	0.01	–	–	80
24°	0.11	0.40	0.33	0.13	0.02	–	–	45
28°	0.13	0.53	0.27	0.07	–	–	–	30
				Group II				
44°	0.07	0.40	0.33	0.13	0.07	–	–	30
40°	0.07	0.40	0.33	0.13	0.07	–	–	30
36°	0.13	0.53	0.27	0.07	–	–	–	30
32°	0.20	0.53	0.27	–	–	–	–	30
28°	0.13	0.40	0.33	0.13	–	–	–	30

Note: This table shows how many intervals of each size were provided by the original schedule and thus known to S-O. Because of inaccuracies in the original settings, gradual changes in the course of the afternoon, and similar factors, the number of intervals, of each size, actually found in the data does not entirely agree with the corresponding number here given.

To read the table: In Group I, 16°, it appears that, out of 40 intervals actually scheduled for Res, 2 (= 0.25) were of 0.00°, 16 (= 0.40) were of 0.05°, and so on.

had become well standardized and proceeded, each pair for itself, in the objective and uniform fashion which work of this type, long conducted and carefully controlled, inevitably assumes; and hence was all the less open to distortion by extraneous factors of this sort.

In brief, the writer believes that he has shown that the procedure remained 'without knowledge' in every essential, despite the minor departures from ordinary practice demanded by his position as both E and O.

Of the seven jars on the table, six were regularly used; each one served in turn as the standard, the other five being then comparison jars; 15 pairs, or temperature intervals, were thus formed: 1-2, 1-4, 1-5, 1-6, 2-3, 2-4, and so on. The seventh jar was kept regularly at 32° (approximate skin temperature of the fingers) and served 1) as adaptation jar in the method of Equivalents and 2) as a reserve jar for use in case one of the others was temporarily out of order. As previously stated, the constants to be used each day were fixed in advance by E-O in accord with a systematic schedule; for example, such a series as the following would be reserved for two successive days:

20.00°
20.00
20.05
20.10
20.15
20.20;

on the first day, the assistant would allocate these values in an order known only to himself among the six jars; the next

any the values were reversely arranged, so that jar 6 had the temperature of jar 1 the day before, jar 5 of jar 2, and so on; exactly the same intervals were then present but in opposed space-order. The above sample set yields 15 intervals, distributed as follows:

0.00°	0.05°	0.10°	0.15°	0.20°
1	5	4	3	2

It is obvious that no possible disposition would give an equal number of intervals for each size; e.g., no arrangement would give three values in each interval above. This being true, the ordinary method by which every interval is compared the same number of times was not feasible. The specific question thus arises: Is it defensible to use more small differences (0.05° and 0.10°) than large ones, with the result that most of the judgments, on which the limens are based, are from small intervals?

In the writer's opinion, the procedure as here used was not only defensible but also preferable. In comparing any two temperatures, we get a certain percentage (p) of W and C judgments; but a p near 0.50 has more effect on the limen than one near 0.00 or 1.00; for example, the "weight" given to p = 0.50 is 1.000, to p = 0.25 or 0.75 is 0.846, to p = 0.05 or 0.95 is 0.352. This being true, it is logical, as Urban[1] points out, to get as many judgments in the region of

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[1]Urban, F.M.: Die Psychophysischen Unsermethoden, etc.; Archiv f.d. Gesamte Psychologie, 16 (1909), p. 182.

P = 0.50 as possible. Against this argument Boring[1] has urged

[1]Boring, R.G.: Urban's Tables, etc.; Amer. Journ. Psych., 28 (1917), p. 282.

two considerations: 1) Large intervals give easy judgments, which have a "steadying effect" on D who is likely to be wearied or confused, apparently, if all comparisons are close and difficult. "What the extreme values lose in mathematical significance, then, is made up by their effect upon the attitude of the observer." 2) If a narrow range of stimuli is used, a small departure of the linen from its expected value will give an asymmetrical distribution of the stimuli about the linen and thus tend to arouse habituation or expectation in O.

As to 1): there were regularly some large differences with correspondingly easy judgments; but there is nothing to prove that a great many of these are needed to maintain the proper attitude in O. In fact, the writer can testify for himself that more than a few obvious judgments in a series were annoying. The only way in which the hundreds of comparisons each afternoon could gain some 'intellectual' interest for him was when they were close enough to be, each time, a real test. When judgments are close, they serve as an incentive to the alert observer; a few easy ones are needed to keep him "steady", but if these are numerous, the judging tends to become oppressively monotonous and 'mechanical'. In this connection let the reader consult Table VI, which gives the range of p_1 and p_2 for each set of data from which a linen was

TABLE VI

Modal and Minimal Percentages of W and G Judgments for Each Set of Data

Position Series

			5°		10°		20°	
			W	G	W	G	W	G
S – J	W	Max.	1.00	1.00	1.00	1.00	1.00	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.00
	G	Max.	1.00	1.00	1.00	1.00	1.00	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.00
B – J	W	Max.			1.00	1.00	1.00	1.00
		Min.			0.00	0.00	0.00	0.00
	G	Max.			1.00	1.00	1.00	1.00
		Min.			0.00	0.00	0.00	0.00

Notes to Table VI: This table gives the modal and minimal percentages – that is, the range – of W and G judgments for every set of data, by the method, from which it was derived.

To read the table: In group 1, time-order S – J (standard preceding comparison), space-order R (comparison to right of standard), 5°, W, it appears that one of the temporal-intervals used gave 0.7% 'worse' judgments, the highest percentage in that set; while at least one other interval gave no 'worse' judgments at all. In 15°, G, likewise, we find that the judgment 'easier' never appeared in at least one interval and always appeared in another.

TABLE VI (Continued)

Group I

			20°		16°		20°		24°		28°	
			W	C	W	C	W	C	W	C	W	C
2-J	R	Max.	0.92	1.00	0.74	1.00	1.00	0.75	0.92	1.00	1.00	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	L	Max.	1.00	1.00	1.00	0.91	0.50	1.00	1.00	0.92	1.00	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00	0.00
J-S	R	Max.	1.00	1.00	1.00	0.84	1.00	0.75	1.00	0.75	1.00	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	L	Max.	1.00	1.00	0.06	0.80	1.00	0.06	1.00	1.00	1.00	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Group II

			44°		40°		36°		32°		28°	
			W	C	W	C	W	C	W	C	W	C
2-J	R	Max.	1.00	1.00	1.00	1.00	1.00	0.75	0.84	1.00	1.00	0.52
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	L	Max.	1.00	0.63	1.00	0.75	1.00	0.01	1.00	0.50	0.69	1.00
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18
J-S	R	Max.	0.75	1.00	0.75	1.00	1.00	1.00	0.75	1.00	1.00	0.92
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	L	Max.	1.00	1.00	1.00	1.00	1.00	1.00	0.75	0.84	1.00	0.92
		Min.	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00

derived. Omitting practice scores, we see that in 48 of 80 cases (= 60 %) p ranged from 0.00 to 1.00; of the remaining 32 (summarized in Table VII), 26 range from 0.70 to 0.99 and 6 are below 0.70. That is, in 60 % of all cases, there were a few differences so obvious that, for example, a w-judgment never occurred ($p_w = 0.00$) with one of them and always occurred ($p_w = 1.00$) with the others; while in 78 of 80 sets (= 93 %), the intervals differed enough to give p a range of 0.70 or more. This may serve to show that while, in general, not very large differences were used, there were nearly always enough to give at least some easy judgments in the set.

TABLE VII

Range of P_w and P_o (excluding 100): Taken from Table VI

		.90 to .99	.80 to .89	.70 to .79	.60 to .69	.50 to .59	.40 to .49	.30 to .39	total
P_w	I	2	1	1		1			5
	II	2	1	4	1				8
P_o	I	2	3	3			1		9
	II	1	2	2	1	1		1	10
Total,		7	7	10	2	2	1	1	32

As to E) above: the point here is, that a reconsideration of an judgment, say E, will tend to make it habitual or expected, and thus artificially raise the value of P_E in that set; hence the stimuli should be so distributed that E and C will, in each set, occur about equally often. Table VIII

TABLE VIII

Percentage Frequencies of W and L Judgments

Group I

			Practice Series		10°	15°	20°	25°	30°
			1st	2nd					
S-J	L	W	0.45	0.47	0.54	0.19	0.22	0.29	0.23
		L	0.55	0.53	0.43	0.54	0.53	0.43	0.57
	R	W	0.51	0.38	0.30	0.38	0.46	0.40	0.34
		L	0.30	0.43	0.45	0.36	0.36	0.32	0.37
J-S	L	W	0.39	0.43	0.39	0.46	0.29	0.40	0.45
		L	0.40	0.34	0.35	0.29	0.32	0.35	0.35
	R	W	0.33	0.26	0.36	0.46	0.57	0.57	0.64
		L	0.53	0.54	0.39	0.26	0.21	0.20	0.20
Arithmetic Mean		W	0.42	0.39	0.37	0.37	0.39	0.42	0.40
		L	0.35	0.42	0.31	0.36	0.36	0.33	0.38

Note: This table gives the percentage-frequencies of W and L judgments for each set of data from which a limen was derived and work; its purpose is to show how 'constant' the several sets, and especially the means of the sets, are in the matter of the relative number of W and L judgments.

To read the Table: In Group I, 15°, S-J (standard preceding comparison jar), 1st (comparison to left of standard), it appears that 0.19 of all the judgments rendered in that set were W and 0.54 were L (the remainder, 0.27, being omitted). The mean of the four W's for this column, 15°, is 0.35 and of the L's is 0.36.

TABLE VIII (Continued)

Group II

			44°	40°	36°	32°	28°
B-S	L	W	0.44	0.62	0.60	0.57	0.35
		C	0.31	0.20	0.16	0.17	0.43
	H	W	0.30	0.53	0.50	0.44	0.48
		C	0.46	0.24	0.14	0.33	0.31
J-S	L	W	0.28	0.33	0.30	0.33	0.40
		C	0.33	0.44	0.42	0.31	0.29
	H	W	0.24	0.27	0.19	0.30	0.35
		C	0.62	0.49	0.56	0.43	0.37
Arithmetic Mean		W	0.32	0.44	0.40	0.41	0.39
		C	0.42	0.34	0.32	0.31	0.36

shows the frequency of W and C for each set. If the four time-space combinations are taken separately, the proportions of W and C sometimes differ widely; consider, as an extreme case, Group I, 28°, J-S, R; 16 % of the judgments were D; of the remainder 64 % were W and 20 % were C. The point of subjective equality[1] here equals -.0.071°; the ex-

[1]The point of 'subjective equality' is defined by Urban (loc. cit., p. 201) as "jene Intensität des Vergleichsreizes, für welche die Wahrscheinlichkeit des Urteiles 'größer' gleich ist der Wahrscheinlichkeit des Urteiles 'kleiner'"; in other words, the point where the two psychometric curves for 'larger' and 'smaller' (in our case, 'warmer' and 'cooler') intersect. This value will be used later; see pp. 106 ff.

cess of stimuli above this point brings a corresponding preponderence of W judgments. But in the same column, opposed time- and space-order, S-J, L, W = 0.23, C = 0.58; the point of subjective equality here being +0.103°, C-judgments predominate. Now even if we assume that the overplus of W's in the first case induced a corresponding bias in E-O's judging, the opposed bias in the other set would tend to neutralize it, when the two are combined in an average. The four time-space combinations had to be used, in each series, with the same set of temperatures; hence, the distribution of stimuli about the point of subjective equality, as the above examples illustrate, was bound to be skewed, now toward W, now toward C, according to the direction of the constant error in each combination. The limen at any level, such as 28°, is the arithmetic mean of the limens for the four time-space orders; hence we must also use the mean of W and C to find out whether the limen has been affected by their relative frequency. By

examining the means, we find, discarding practice series, that W exceeds C in seven cases, equals C in two, and falls below it in one. The difference never exceeds 0.10, except at 44° which is anomalous both in extent and direction of difference. The writer does not believe that such small inequalities in the frequencies of W- and C-judgments could have any appreciable effect on the means, but knows no way of proving or disproving it from the data. It may be of value to note that, in Urban's extensive study of lifted weights, conducted with every precaution and under conditions which may be considered ideal, appear the following percentages of Heavier and Lighter judgments for his seven C's at the one level that one used (100 gr.):[1]

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[1] Urban, F.M.: Die psychophysischen Massmethoden, etc.; Archiv.f.d. ges. Psych., 15, (1909), p. 257.

	I	II	III	IV	V	VI	VII
H	0.37	0.38	0.38	0.42	0.46	0.33	0.33
L	0.41	0.47	0.57	0.45	0.44	0.46	0.49

The mean of the differences, disregarding sign, is 10.1; the same mean for our data (Table VIII), practice series omitted, is 6.2. Our results seem, therefore, to compare favorably with Urban's in this respect.

The temperature having been properly set and checked, as detailed above, the process of observing was ready to begin. Almost all the observations in these data were taken between the hours of noon and five p.m., one to five being the regular 'working-period' each day. The judgments were re-

carded by assistants, most of whom were secured from the writer's classes. A cardboard stencil had been prepared to fit the page of the record-book and pierced with 120 holes, arranged in six sections of 20 each; through these openings the judgments were recorded on the page beneath. The sections were numbered from I to VI, to tally with the six jars. The five openings in the first row, Section I, were reserved and marked for the five possible combinations of jar I with the others, in regular order, viz.: 1-2, 1-3, 1-4, 1-5, 1-6. The other three rows were duplicates of the first except for the order in which the combinations were taken and the comparisons made; in each row the judgments were taken and recorded in a different, but fixed, order, as follows:

First:	1-2,	1-3,	1-4,	1-5,	1-6,
Second:	1-4,	1-2,	1-6,	1-3,	1-5,
Third;	1-3,	1-2,	1-5,	1-4,	1-6,
Fourth:	1-5,	1-2,	1-4,	1-3,	1-6.

By this device any two jars were compared four times in a full series, but each time preceded by a different pair; for example, the comparison of 1-6 was preceded, in the first row, by 1-5, in the second by 1-2, in the third by 1-4, in the fourth by 1-3. The same plan was used in the other sections. This arrangement was designed to make innocuous the influence that any judgment is likely to have on the one immediately following. It is well-known, for example, that if the preceding variable was distinctly cooler than the standard, the next one, slightly cooler, may be judged doubtful or even warmer, simply because the difference is less conspicuous than the

preceding; whereas apart from this influence the judgment might have been, correctly, equal. If the one or two differences preceding have been small, on the contrary, the judgment is more likely to be accurate. The above arrangement served to obviate any uniform effect of this sort.

Provided with this stencil, the recorder, who sat at a table nearby, had a simple task. He read the combinations in their stated order and as each one was called, O gave his judgment which was then recorded through the proper opening on the page beneath. At the end of a series (covering about 45 minutes), 120 judgments had thus been made and recorded.

As above stated, every combination, or pair, of jars was judged four times in each order in the course of a series; for example, jars 1-2 were compared four times in that order and also four times in reverse order, 2-1; eight comparisons for each of the fifteen combinations that make up the total of 120 judgments in a series.[1] In addition, three to four

[1]In comparing jars 1-2 (1 being the standard), O gave his judgment in terms of 2; in the reversed order 2-1 (2 now being standard), the judgment applied to 1. For example, in the former case, O judged that 2 is heavier than 1, in the latter that 1 is heavier than 2.

series were regularly completed in an afternoon. It will at once be seen that this procedure departs from the customary one, in which the stimuli are presented to O in such wise that he cannot possibly know, when comparing any two, what judgment he gave to the same pair before. For example, in Brown's study, the weights were arranged in an order unknown to O

c on a rotating table which was screened from his view; it was thus easy to keep him from knowing with which pair of weights he was dealing or what judgment he had given them estimating them before. Any such control was out of the question under the necessary conditions of our experiment. The bulky and elaborate mechanism precluded any manipulation of the jars and the temperature-intervals, of course, remained constant; so that, in most cases, E-O could not avoid remembering, as he repeated a given comparison four times successively in a series, what estimate he had previously given to that same pair. The question therefore arises and demands careful attention: Did the fact that E-O compared what he knew to be the same interval as often as 16 times in an afternoon, in each time-order, with no change in the original setting, give evidence of modifying the later judgments? Several things, for instance, might be expected to happen: In the case of a judgment originally D, repetition might refine it to the correct one, W or C, and thus artificially decrease the limen below what it would be if E-O had no knowledge of previous trials; or, remembering how a given interval had been rated before, E-O might allow himself merely to repeat the same judgment in later estimates, instead of making a fresh and careful scrutiny each time; or, conversely, to avoid mechanical repetition, E-O might tend to switch, arbitrarily, from one judgment to another in successive trials. To answer this question, which is fundamental for the worth of our data and the accuracy of our results, a careful survey of all the material on which our calculations are based was made.

The results of this survey are exhibited in the following tables, Tables I to XII. The main percentage-frequencies of right judgments in the four successive trials of each series, at all temperatures, are shown in Table IX, and the same values for wrong judgments in Table X. These figures are tabulated for this reason: If O has any tendency to refine or improve his later judgments as a result of having employed the rule just a number of times before, it may be expected to emerge either as an increase of right, or as a decrease of wrong, judgments; if there is a tendency to exaggerate, the opposite effect may also appear. If the proper addition of these data, however, reveals significant or uniform changes from earlier to later trials, we may assume that such tendency has been operative; on the other hand, if neither of these two criteria can be made to yield instances of such uniform or significant changes, we are then, in the writer's opinion, entitled to deny any appreciable influence upon our data from this source.

Other variables, such as the time-error series, also affect the number of right and wrong judgments; hence only those figures can legitimately be compared in which these figures are constant. The status of the values recorded in Tables IX and X, with reference to each other, is as follows: 1) The four successive figures in any row of any section are based on strictly uniform conditions, except for the order in which they stand. For example, section A_1, I, 24°, the frequency of right judgments in the four successive trials was: 0.62, 0.69, 0.73, 0.69; these are directly comparable in all respects,

Notes to Tables IX and X

Table IX is designed to show the mean percentage-frequencies of Right judgments for the four successive trials in each series at all temperatures from which items are used in our results. By a "right" judgment is meant one that agrees with the objective difference; e.g., with an interval of +0.[illegible]°, G is right and with one of − 0.1[illegible]°, S is right.

In an afternoon four full series were usually completed, here represented by I, II, III, IV. The temperatures are grouped into three sections: those above 25°, those below 25°, and 25° itself. The letters A_1, A_2, and so on, distinguish the sections for reference and comparison.

In covering a series, two jars would first be compared in one order, say 3 – 5, and then after in the reverse order, 5 – 3. The first or 'original' order is tabulated in sections A_1, B_1, and C_1, and the following or 'reverse' order in A_2, B_2, and C_2. A comparison of A_1 with A_2, and so on, will thus reveal whether the correctness of the reverse judgments was affected by the preceding originals.

Consider section A_1, 16°, where 0.71 appears in the first column of series I. At the beginning of an afternoon's work, none of the intervals had as yet been examined before; each of the fifteen combinations was, of course, compared for the first time in series I. Of these 15 beginning judgments in series I of each day, for the whole time devoted to 16°, 0.71 were right. The second time through 0.[illegible] were right, the third time 0.86, the fourth time, 0.86. Series II, following within 10 to 15 minutes ordinarily, began with 0.71 right, then sank to 0.67, and so on. The reverse comparisons, in each case, follow directly upon the originals shown in the same column, above. In A_2, 16°, 0.50 of the reverse judgments, directly after 0.71 above, were right; those immediately succeeding 0.[illegible] above were 0.[illegible] right, and so on.

The last row, in A_1, A_2, C_1, and C_2, gives the means of the four values directly above; the figure in the lower right corner of each section-series is the mean of all values therein; e.g., in A_1, I, 0.6[illegible] is the mean of the 16 individual values. D_1, in the first four columns of each series, gives the means of the 'original' values in A_1, B_1, and C_1; D_2 of the 'reverse' values in A_2, B_2 and C_2. The m.d. for all these values was calculated by the usual formula:

$$\left[\frac{\Sigma d^2}{N(N-1)}\right]^{\frac{1}{2}} .$$

The extreme right figure for each series in D_1 and D_2 (e.g., 0.650 in D_2, I) is a general mean derived from the four pre-

ending ones in the same row; at the extreme right of the page appears the mean for each section as a whole; it is derived from the means of the four series in that section. The s.d. for these general means was found by the formula:

$$\text{s.d.}\,\varepsilon_{\bar{x}}(x_1, x_2, \ldots, x_n) = \frac{1}{n^2}(\text{s.d.}\,\varepsilon_{x_1} + \text{s.d.}\,\varepsilon_{x_2} + \ldots + \text{s.d.}\,\varepsilon_{x_n}).$$

This should be noted, since a direct computation of s.d. from the component values of the mean will sometimes give a figure differing by one or two thousandths.

It is to be noted that judgments applying to 0.00°-intervals are omitted from the table; for the reason that the only judgment that can be considered right here is D, both U and S being wrong. Hence these figures are not strictly comparable with the others.

The above notes apply also to Table X, which exhibits the percentage-frequencies of Wrong judgments in the same way.

TABLE IX

Mean Percentage Frequencies of Right Judgments

Series		I				Group Average	II				Group Average
A_1	20°	0.66	0.[illegible]	0.75	0.75		0.73	0.64	0.76	0.[illegible]	
	16°	0.71	0.[illegible]	0.46	0.56		0.71	0.57	0.67	0.60	
	20°	0.72	0.67	0.52	0.61		0.48	0.64	0.[illegible]	0.[illegible]	
	24°	0.[illegible]	0.69	0.[illegible]	0.69		0.6[illegible]	0.[illegible]	0.[illegible]	0.[illegible]	
M_{A_1}		0.[illegible]	0.70	0.58	0.65	0.65	0.[illegible]	0.60	0.[illegible]	0.62	0.62
P.E.		.0[illegible]	.0[illegible]	.063	.0[illegible]	.0[illegible]	.057	.0[illegible]	.049	.0[illegible]	.0[illegible]
A_2	20°	0.74	0.76	0.70	0.[illegible]		0.58	0.73	0.70	0.70	
	16°	0.50	0.54	0.50	0.63		0.57	0.74	0.74	0.71	
	20°	0.68	0.79	0.79	0.61		0.74	0.71	0.71	0.[illegible]	
	24°	0.65	0.69	0.85	0.58		0.73	0.65	0.73	0.[illegible]	
M_{A_2}		0.[illegible]	0.70	0.71	0.66	0.68	0.66	0.71	0.72	0.67	0.69
P.E.		.0[illegible]	.056	.078	.049	.0[illegible]	.046	.020	.003	.0[illegible]	.0[illegible]
B_1	20°	0.67	0.[illegible]	0.67	0.73		0.70	0.65	0.65	0.65	
		0.70	0.74	0.78	0.70		0.[illegible]	0.[illegible]	0.67	0.[illegible]	
						0.73					0.73
						.0[illegible]					.0[illegible]
B_2	20°	0.[illegible]	0.[illegible]	0.[illegible]	0.[illegible]		0.75	0.75	0.75	0.75	
		0.70	0.[illegible]	0.70	0.[illegible]		0.67	0.[illegible]	0.[illegible]	0.[illegible]	
						0.70					0.73
						.0[illegible]					.0[illegible]

TABLE IX

For Rock Series at Rock Temperature

III	Group Average	IV	Group Average	General Average
0.72 0.55 0.66 0.61		0.63 0.63 0.69 0.61		
0.52 0.46 0.61 0.61		0.50 0.75 0.50 0.75		
0.61 0.76 0.71 0.57		0.75 0.75 0.73 1.00		
0.55 0.91 0.73 0.64		0.64 0.82 0.73 0.64		
0.60 0.68 0.68 0.66	0.65	0.63 0.74 0.66 0.80	0.71	0.695
.044 .098 .027 .053	.029	.051 .039 .055 .075	.031	0.014
0.67 0.76 0.66 0.72		0.69 0.50 0.44 0.50		
0.69 0.61 0.74 0.65		0.88 0.82 0.75 0.88		
0.49 0.67 0.67 0.76		0.58 0.92 0.83 0.64		
0.82 0.45 0.85 0.91		0.82 0.72 0.64 0.91		
0.66 0.62 0.66 0.76	0.68	0.74 0.76 0.67 0.73	0.72	0.693
.068 .065 .040 .056	.029	.067 .095 .085 .095	.040	0.015
0.47 0.82 0.71 0.71		0.77 0.73 0.57 0.70		
	0.68		0.69	0.708
	.074		.043	0.023
0.41 0.41 0.47 0.39		0.57 0.77 0.73 0.83		
	0.47		0.73	0.668
	.042		.006	0.013

TABLE IX (Continued)

Series		I				Group Average	II				Group Average
C_1	144°	0.56	0.59	0.44	0.50		1.00	0.91	0.75	0.91	
	108°	0.75	0.69	0.50	0.69		0.67	0.71	0.71	0.62	
	36°	0.63	0.63	0.50	0.50		0.81	0.65	0.47	0.59	
	72°	0.76	0.65	0.71	[illegible]		0.79	0.82	0.63	[illegible]	
$M.C_1$		0.68	0.64	0.54	0.58	0.61	0.72	0.67	0.69	0.76	.69
P.E.		.048	.003	.038	.118	.024	.103	.101	.061	.072	.053
C_2	144°	0.61	0.58	0.96	0.69		0.33	0.58	0.58	0.50	
	108°	0.75	0.60	0.67	0.75		0.58	0.61	0.62	0.71	
	36°	0.67	0.67	0.67	0.53		0.72	0.65	0.59	0.59	
	72°	0.69	0.61	0.53	0.59		0.79	0.79	0.80	0.68	
$M.C_2$		0.71	0.65	0.70	0.64	0.67	0.59	0.72	0.67	0.64	.65
P.E.		.048	.054	.032	.049	.027	.103	.035	.074	.030	.029
General Mean (D_1)		0.67	0.67	0.59	0.66	0.65	0.69	0.66	0.64	0.69	0.67
P.E.		.030	.027	.040	.048	.018	.053	.044	.029	[illegible]	.032
General Mean (D_2)		0.69	0.70	0.71	0.65	[illegible]	0.64	0.72	0.71	0.65	0.680
P.E.		.032	.021	.045	.032	.019	.054	.035	.031	.028	.016

TABLE IX (Continued)

III				Group Average	IV				Group Average	General Average
0.77	0.77	0.71	0.45		0.60	0.60	0.70	0.70		
0.79	0.89	0.89	0.89		0.58	0.58	0.55	0.50		
0.80	0.70	0.55	0.60		0.64	0.61	0.61	0.71		
0.68	0.68	0.58	0.64		0.78	0.63	0.56	0.74		
0.76	0.76	0.71	0.65	0.72	0.65	0.66	0.61	0.66	0.64	0.665
.027	.047	.070	.091	.031	.045	.053	.034	.055	.022	0.017
0.74	0.65	0.61	0.71		0.80	0.90	0.70	0.80		
0.58	0.53	0.53	0.63		0.87	0.87	0.87	0.67		
0.65	0.75	0.75	0.70		0.50	0.67	0.72	0.43		
0.60	0.52	0.52	0.76		0.48	0.56	0.48	0.52		
0.64	0.61	0.60	0.70	0.64	0.66	0.75	0.69	0.61	0.68	0.660
.036	.053	.053	.027	.021	.102	.081	.080	.082	.041	0.016
0.66	0.73	0.69	0.66	0.688	0.65	0.70	0.65	0.73	0.678	0.671
.041	.048	.031	.028	.021	.032	.030	.029	.046	.017	0.0098
0.68	0.59	0.61	0.71	0.633	0.69	0.76	0.68	0.69	0.705	0.677
.041	.042	.034	.031	.019	.053	.051	.045	.059	.027	0.104

TABLE 2

Mean Percentage Frequencies of Wrong Judgments

Series		I				Group Average	II				Group Average
A_1	20°	0.12	0.0[illegible]	0.03	0.03		0.06	0.06	0.00	0.03	
	16°	0.04	0.15	0.13	0.08		0.0[illegible]	0.14	0.20	0.05	
	20°	0.18	0.05	0.14	0.18		0.22	0.11	0.18	0.20	
	24°	0.11	0.04	0.10	0.04		0.15	0.23	0.23	0.12	
$M._{A_1}$		0.12	0.08	0.11	0.08	0.097	0.12	0.14	0.15	0.12	0.124
P.E.		.029	.021	.025	[illegible]	.013	[illegible]	[illegible]	[illegible]	[illegible]	.020
A_2	20°	0.0[illegible]	0.05	0.0[illegible]	0.03		0.06	0.06	0.06	0.03	
	16°	0.17	0.19	0.17	0.[illegible]4		0.09	0.09	0.09	0.04	
	20°	0.11	0.07	0.00	0.12		0.08	0.00	0.15	0.05	
	24°	0.13	0.04	0.04	0.04		0.08	0.02	0.19	0.04	
$M._{A_2}$		0.13	0.11	0.06	0.06	0.07	0.07	0.06	0.12	0.0[illegible]	0.0[illegible]
P.E.		[illegible]	.0[illegible]1	[illegible]	.019	.020	.015	.010	[illegible]	[illegible]	.011
B_1 (1)	22°	0.05	0.05	0.10	0.07		0.17	0.10	0.10	0.05	
B_1 (2)	22°	0.13	0.0[illegible]	0.08	0.0[illegible]	[illegible]	0.00	0.17	0.17	0.00	
M.						0.08					0.0[illegible]
P.E.						[illegible]					[illegible]
B_2		0.05	0.10	0.05	0.08		0.05	0.00	0.10	0.05	
		0.09	0.09	0.13	0.0[illegible]		0.00	0.0[illegible]	0.00	0.17	
M.						[illegible]					0.066
P.E.						.00[illegible]					[illegible]

TABLE X

For Each Series at Each Temperature

III	Group Average	II	Group Average	General Average
0.06 0.17 0.07 0.05		0.00 0.00 0.00 0.00		
0.04 0.13 0.13 0.04		0.00 0.13 0.13 0.25		
0.12 0.04 0.04 0.08		.00 0.00 0.00 0.00		
0.00 0.00 0.00 0.00		0.00 0.00 0.16 0.09		
0.06 0.09 0.06 0.05	0.063	0.08 0.03 0.08 0.09	0.049	0.053
.025 .039 .027 .019	.013	.000 .033 .046 .051	.020	0.009
0.05 0.03 0.00 0.00		0.00 0.13 0.25 0.00		
0.09 0.17 0.04 0.09		0.00 0.00 0.00 0.00		
0.16 0.12 0.04 0.16		0.08 0.00 0.00 0.00		
0.00 0.00 0.16 0.00		0.00 0.00 0.00 0.00		
0.07 0.08 0.07 0.06	0.069	0.02 0.03 0.06 0.00	0.29	0.064
.035 .041 .040 .039	.019	.020 .033 .063 .000	.017	0.009
0.00 0.06 0.10 0.15		0.10 0.10 0.13 0.03		
	0.105		0.090	0.085
	.055		.021	0.014
0.00 0.12 0.06 0.00		0.03 0.03 0.00 0.07		
	0.045		0.033	0.052
	.029		.014	0.010

TABLE X (Continued)

Series		I				Group Average	II				Group Average
C_1	44°	0.19	0.11	0.26	0.22		0.00	0.05	0.00	0.00	
	40°	0.13	0.06	0.06	0.06		0.19	0.10	0.00	0.00	
	36°	0.13	0.13	0.13	0.13		0.06	0.12	0.10	0.10	
	32°	0.00	0.00	0.06	0.00		0.00	0.11	0.00	0.00	
M_{C_1}		0.11	0.05	0.15	0.10	0.104	0.06	0.10	0.05	0.05	0.064
P.E.		.040	.039	.047	.047	.019	.048	.009	.043	.043	.013
C_n	44°	0.00	0.00	0.00	0.00		0.05	0.25	0.00	0.25	
	40°	0.13	0.19	0.25	0.13		0.10	0.00	0.00	0.00	
	36°	0.07	0.07	0.07	0.07		0.18	0.12	0.06	0.06	
	32°	0.00	0.06	0.06	0.00		0.11	0.00	0.11	0.11	
M_{C_2}		0.05	0.08	0.10	0.05	0.069	0.16	0.09	0.06	0.11	0.105
P.E.		.031	.040	.058	.031	.019	.035	.060	.023	.053	.022
General Mean (D_2)		0.11	0.07	0.11	0.26	0.092	0.06	0.12	0.10	0.07	0.093
P.E.		.019	.014	.021	.023	.010	.025	.015	.029	.022	.013
General Mean (D_3)		0.06	0.09	0.06	0.06	0.079	0.10	0.06	0.08	0.06	0.079
P.E.		.021	.027	.025	.010	.011	.023	.026	.010	.004	.011

TABLE X (Continued)

III				Group Average	IV				Group Average	General Average
0.06	0.08	0.10	0.19		0.10	0.20	0.00	0.20		
0.11	0.11	0.05	0.11		0.15	0.06	0.19	0.15		
0.00	0.00	0.10	0.05		0.06	0.11	0.08	0.06		
0.00	0.08	0.08	0.08		0.04	0.00	0.04	0.00		
0.04	0.06	0.08	0.11	0.074	0.08	0.09	0.08	0.10	0.088	0.083
.027	.023	.012	.030	.012	.020	.042	.041	.043	.017	0.009
0.06	0.16	0.16	0.08		0.10	0.00	0.10	0.10		
0.11	0.16	0.11	0.00		0.05	0.00	0.00	0.00		
0.05	0.05	0.00	0.05		0.06	0.05	0.11	0.00		
0.08	0.08	0.08	0.08		0.19	0.19	0.15	0.07		
0.08	0.11	0.09	0.10	0.095	0.10	0.06	0.09	0.04	0.071	0.084
.013	.028	.034	.050	.017	.035	.041	.032	.025	.017	0.009
0.04	0.07	0.08	0.09	0.072	0.05	0.07	0.08	0.08	0.071	0.082
.014	.019	.018	.020	.009	.017	.024	.020	.023	.013	0.0057
0.06	0.10	0.07	0.07	0.077	0.05	0.04	0.07	0.05	0.02	0.071
.017	.021	.022	.030	.011	.021	.023	.030	.019	.011	0.0095

care for the possible effect of preceding trials. The same is true of the four column-means in any section-series of the table. 2) When comparing original with the next following reverse frequencies (for example, in A_1 and A_2, I, 20°, first column, the figures 0.62 and 0.65), we must note a) that the space-order also reverses; and b) that the standard (S) and comparison (J) jars are interchanged. For example, if, in the original comparison, we follow the order 1 – 2, jar 1 being S and 2 being J, we have, in the reverse procedure, the order 2 – 1, 2 now being S and 1 being J. The time-order thus remains constant throughout the series in relation to S, but we actually compare the same jars, by the two procedures, in reversed time-orders. In general, these orders would have opposed constant errors, which would affect the number of right judgments. These values therefore cannot properly be compared. 3) The means of successive series in the same section (e.g., A_2, I and II, the values 0.62 and 0.69) are not comparable, since the combination of time- and space-orders, as well as the number of cases from each series, vary from one series to the next in the table; even though, as above indicated, they are constant within the same series. 4) For the same reason, as in 3), values from different temperatures (e.g., A_1 and C_1) may not be compared.

In Tables XI and XII is summarized the principal evidence, that can be gleaned from Tables IX and X, which bears on our two-fold question: Do the data reveal any changes, uniform in direction or significant in size, from earlier to later frequencies in the same series?

TABLE XI

Table of Maximal Changes

Between any Two Adjacent Column-Means

Same Section and Series (From Tables IX and X)

	Rights						Wrongs					
	Inc.	s.d.	Inc./s.d.	Inc.	s.d.	Inc./s.d.	Inc.	s.d.	Inc./s.d.	Inc.	s.d.	Inc./s.d.
A_1	.14	.093	1.5	.12	.083	1.4	.05	.056	0.9	.04	.025	1.1
A_2	.10	.068	1.5	[.05 / .09]	[.023 / .127]	[2.2 / 0.7]	.06	.035	1.7	.08	.028	2.9
U_1	.10	.101	1.0	.10	.063	1.6	.05	.055	0.9	.05	.046	1.1
U_2	.12	.117	1.0	.08	.115	0.7	.05	.058	0.9	.07	.069	1.0
D_1	.10	.054	1.9	.10	.048	2.1	.04	[.025 / .029]	[1.6 / 1.4]	.04	.024	1.7
D_2	.10	.046	2.2	.08	.070	1.1	.04	.027	1.5	.04	[.035 / .033]	[1.1 / 1.2]

Notes: In Table XI is recorded the largest change, both increase and decrease, to be found in every section of Tables IX (Rights) and X (Wrongs); as well as every change, for the same data, that equals or exceeds 2 s.d. The derivation and import of the values will be clear from an example: In Table IX, section A_2, we find, in the largest inc., 4 column-means in each section-series. If we compare each mean with the one next following in the same series, we have in A_2, I, these changes: + 0.06 (0.64 to 0.70), + 0.01 (0.70 to 0.71), − 0.05 (0.71 to 0.66); the next series begins with + 0.05 (0.66 to 0.71), and so on. Of the 12 changes in that section, A_2, the maximal decrease, as recorded in the present table, is − 0.09 (IV, 0.76 to 0.67); its s.d. is 0.127, so that the decrease is 0.7 times its s.d. We find here also a decrease of 0.05 (II, 0.72 to 0.67) which is less than the other, but, owing to its small s.d. (0.023), is 'significant' (= 2.2 s.d.). When the maximal value occurs twice, the s.d. for each instance is given.

No column-means were formed for D_1 and D_2, which are therefore omitted from our tabulation.

Notes to Table XII: This table is also compiled from Tables IX and X; it is designed to exhibit the number of changes - positive, negative and zero - that occur from one column to the next within the same section and series, without reference to the size of these changes.

To understand the origin of these figures, consider Table IX, section A_1, series I. In the first row, after 20°, we find an increase in the percentage of right judgments from the first set of comparisons to the second (0.66 to 0.84), a decrease from second to third (0.84 to 0.75), and no change from third to fourth (0.75 and 0.75). These three cases are recorded in the present table under Part I, Rights, I, A_1; one in each of the columns +, 0, -. If we now compare each percentage with all that follow it within the same series instead of only the next one, we find, in A_1, I, 20°, two increases (0.66 to 0.75) and one decrease (0.84 to 0.75), in addition to the three above. By this method each row yields just twice as many changes as before; so that, in Part I, $A_1 + B_1 + C_1 = 30$ (13 plus, 3 zero, 14 minus), in Part II, = 60 (20 plus, 7 zero, 33 minus). There being 10 cases (that is, rows), - 4 in A_1, 2 in B_1, 4 in C_1 - the plus and minus means in the latter example are 2.00 and 3.30, with s.d.'s of 0.42 and 0.45, respectively. The difference of these two means is 1.30, with s.d. of 0.62 (in the section Difference (+ & -)M_1).

The figures under 'Rights' are from Table XI, those under 'Wrongs' from Table X.

TABLE XII

Successive Changes in Percentage-Frequency

Within the Same Series (From Tables IX and X)

Part I: Rights

	I			II			III			IV		
	+	0	-	+	0	-	+	0	-	+	0	-
A_1	5	1	6	5	0	7	5	1	6	6	2	4
A_2	7	1	4	3	3	6	7	1	4	4	1	7
B_1	4	0	2	1	3	2	1	2	1	1	0	2
B_2	2	2	2	1	4	1	2	1	0	2	0	1
C_1	4	2	6	6	1	5	2	5	5	4	3	5
C_2	4	2	6	4	4	4	4	3	5	6	2	4
$A_1 + B_1 + C_1$	13	3	14	12	4	14	8	7	12	11	5	11
$Mean_1$	1.30	0.30	1.40	1.20	0.40	1.40	0.89	0.78	1.33	1.22	0.56	1.22
S.D. (M)	0.21		0.16	0.20		0.16	0.20		0.24	0.22		0.28
$A_2 + B_2 + C_2$	13	5	12	8	11	11	13	5	9	12	3	12
$Mean_2$	1.30	0.50	1.20	0.80	1.10	1.10	1.44	0.56	1.00	1.33	0.33	1.33
S.D. (M)	0.21		0.20	0.20		0.23	0.18		0.24	0.24		0.17
Difference (+ & -) M_1		0.10			0.20			0.44			0.00	
S.D. (Diff.)		0.27			0.26			0.31			0.36	
Difference (+ & -) M_2		0.10			0.30			0.44			0.00	
S.D.(Diff.)		0.29			0.31			0.29			0.29	

TABLE XII

Successive Changes in Percentage-Frequency

Within the Same Series (From Tables IX and X)

Part I: Wrongs

	I			II			III			IV		
	+	0	-	+	0	-	+	0	-	+	0	-
A_1	4	2	6	3	2	5	4	3	3	3	5	1
A_2	2	5	3	2	5	5	4	3	3	2	5	2
B_1	1	2	3	1	3	2	2	1	0	1	1	1
B_2	3	1	2	2	2	2	1	0	2	1	1	1
C_1	2	6	4	4	4	4	3	3	2	3	0	7
C_2	3	7	2	2	5	5	4	5	3	2	4	6
$A_1 + B_1 + C_1$	7	10	13	10	9	11	11	11	5	5	13	9
$Mean_1$	0.70	1.00	1.30	1.00	0.90	1.10	1.22	1.22	0.56	1.00	1.00	1.00
S.D. (M)	0.21		0.21	0.21		0.13	0.22		0.18	0.24		0.29
$A_2 + B_2 + C_2$	8	13	9	6	12	12	9	8	10	9	9	9
$Mean_2$	0.80	1.30	0.90	0.60	1.20	1.20	1.00	0.89	1.11	0.56	1.44	1.00
S.D. (M)	0.25		0.23	0.16		0.20	0.24		0.26	0.24		0.24
Difference (+ 2 -) M_1		0.60			0.10			0.66			0.00	
S.D. (Diff.)		0.30			0.26			0.26			0.37	
Difference (+ 2 -) M		0.10			0.60			0.11			0.44	
S.D. (Diff.)		0.34			0.26			0.35			0.34	

TABLE XII (Continued)

Part II: Nights

	I			II			III			IV		
	+	0	-	+	0	-	+	0	-	+	0	-
$A_1 + B_1 + C_1$	20	7	33	22	10	28	20	9	25	23	8	23
$Mean_1$	2.00	0.70	3.30	2.20	1.00	2.80	2.22	1.00	2.78	2.56	0.89	2.56
S.D. (M_1)	0.42		0.45	0.51		0.33	0.52		0.57	0.50		0.65
$A_2 + B_2 + C_2$	24	11	25	18	16	26	32	6	16	21	9	24
$Mean_2$	2.40	1.10	2.50	1.80	1.60	2.60	3.56	0.67	1.78	2.33	1.00	2.67
S.D. (M_2)	0.40		0.37	0.47		0.56	0.34		0.40	0.50		0.29
Difference (+ & -) M_1		1.30			0.60			0.56			0.00	
S.D. (Diff.)		0.62			0.61			0.77			0.62	
Difference (+ & -) M_2		0.10			0.80			1.78			0.34	
S.D. (Diff.)		0.55			0.73			0.52			0.58	

Part II: Errors

$A_1 + B_1 + C_1$	17	19	24	16	16	26	24	18	10	20	20	14
$Mean_1$	1.70	1.90	2.40	1.60	1.60	2.60	2.89	2.00	1.11	2.22	2.22	1.56
S.D. (M_1)	0.54		0.52	0.51		0.41	0.59		0.39	0.57		0.44
$A_2 + B_2 + C_2$	13	30	17	14	22	24	15	19	20	10	24	20
$Mean_2$	1.30	3.00	1.70	1.40	2.20	2.40	1.67	2.11	2.22	1.11	2.67	2.22
S.D. (M_2)	0.46		0.45	0.43		0.43	0.50		0.52	0.45		0.57
Difference (+ & -) M_1		0.70			0.80			1.78			0.66	
S.D. (Diff.)		0.75			0.66			0.70			0.72	
Difference (+ & -) M_2		0.40			1.00			0.55			1.11	
S.D. (Diff.)		0.60			0.60			0.72			0.73	

In Table XI are recorded the serial changes to be found in each of the groups of data represented, as well as every change, for these same data, which equals or exceeds 2 s.d.[1] If we now assume that any difference equal to or exceeding twice its s.d. is 'significant', we can see at once, from this tabulation, how many significant changes occur among all the comparable means of Tables IX and X. There are 72 of these actual changes in Table IX (Rights) and 72 in Table X (Wrongs); 12 for each of the six sections included in Table XI. Out of this number we find significant ones to the following extent:

	Number of changes	Increases	Percent	Decreases	Percent
Rights,	72	1	1.4	2	2.8
Wrongs,	72	0	0.0	1	1.4

It will be noted that three Rights and one Wrong are significant;[2] the percent ranging, for each category, from 0.0 to 2.8. In brief, these figures show: 1) a very small percentage of significant changes; 2) in both Rights and Wrongs

[1] Two s.d., which equals about three p.e., is commonly taken as the lowest acceptable criterion of significant value; conservative workers usually demand a higher figure.

[2] It will be recalled that these changes include only such as occur from any mean to the one next following in a series, e.g., in Table IX, A_1, I, are the means 0.63, 0.70, 0.58, 0.65, showing, in order, increase, decrease, increase. If we now compare each mean with all that follow (e.g., 0.63 and 0.58, 0.63 and 0.65, 0.70 and 0.65) we get six changes instead of three; that is, 144 in all for both Rights and Wrongs instead of 72. It is of interest that, including these additional changes, we get exactly the same percentage of cases in each of the four categories as we find in the table above.

the 'decreases' have a slight preponderance; what evidential value this fact would seem to be is the negative; for the reason that a given tendency, operating upon two factors, Rights and Errors, whose frequencies tend roughly to rise and fall inversely, would hardly show both of them in the same direction;[1] 3) in the Rights, significant changes in both direc-

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[1] A tendency to give more I-judgments toward the close of a series would indeed have this effect, and would reasonably be opposed by fatigue and decrease of interest and attention; but the preponderance is so very slight as to be of no significance.

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tions appear; these would hardly both be due to the notion of a common factor; 4) the fact that only decreases occur in the Errors might be accepted as establishing the presence of a tendency to refine and improve the later judgments of a series, were not the number of these decreases quite negligible. In short, the writer believes that no evidence of any consistent or significant influence can be derived from the data of Table XI.

We turn now to Table XII, which exhibits the number of changes - positive, negative and zero - that have occurred in the same series; with no reference to the size of those changes. The question to be answered by this collation of figures may be stated thus: Do we find a significant difference between the number of plus and minus changes, as we follow the successive judgments of a series, such as would prove a tendency for later judgments to become more or less accurate than the earlier ones? For example, Setter IV, As, III, after

the second 50", we find the right judgments, in the four successive trials of that series, to equal: 0.49, 0.67, 0.67, 0.84. We have here an increase, then no change, then another increase; so that the final set of judgments is distinctly more accurate than the first. May we accept from the data as a whole some written summary of this sort for later estimates to become more (or less) accurate? Obviously, a significant correspondence of positive changes could imply, in the case of right, that the later judgments were more accurate, while an excess of negative changes would argue the reverse. Next, in Table XII, the reader is asked to consider the differences between the number of plus and minus changes in each series. How these differences are determined is explained in the notes to the table, page 63. If we now examine those in Part I we find, among the rights, none that met our criterion of significance (Difference = 2 S.D.$_{difference}$); among the lefts, page 61, we find three: Sessions A and B, Series I, 0.60 (S.D. = 0.10), Series II, 0.60 (S.D. = 0.26), Series III, 0.66 (S.D. = 0.08). In Part II, page 66, we find, Rights, Series I, 1.30 (S.D. = 0.61), Series III, 1.71 (S.D. = 0.31); Lefts, Series III, 1.71 (S.D. = 0.70). In tabular form:

	Rights		Lefts	
Larger loss,	Plus	Minus	Plus	Minus
Part I,	0	0	1	2
Part II,	1	1	1	0

Three points may be noted:

1) If there is a real tendency for judgments to become more

(or less) accurate as a series proceeds, we might, in general, expect it to appear in both Lights; that is, by either method of tabulation. Now, on the contrary, in only one case do we find the corresponding value in the other part large enough to be significant (Bronze, III); in the remaining four this is not true.

2) No uniform tendency can be detected at any point. Plus and minus are in excess equally often, in both Lights and Bronze. The area factor, in general, could hardly increase now the plus and now the minus value, in both Lights and Bronze, which tend to vary inversely. A decrease in the number of D's might indeed have an effect of this sort under certain possible conditions of the constant error; but see 3) below.

3) In Part II, Series III, the plus mean exceeds by 1.7% in both Lights and Bronze. Ordinarily, if these excesses of excess rose due to a common cause, it would not bias both Lights and Bronze in the same direction. However, as in 2) above, a decrease in the frequency of D's, under certain conditions of the constant error, may have the effect of producing more Bronze in the original comparisons and more Lights among the corresponding reverses; so we find in these two values near the ends series, III. We might thus be inclined to assume, here and in 2), a decline in D's and thereby a progressive refinement of judging as the series proceeds. Such an increase seems undesirable, however, because the general trend in the remaining series, if we take the differences as they stand, is just the reverse - the decreases predominate rather decidedly

in both sights and tones; that is, the I-judgments tend to increase slightly as the series proceeds. Consider this table:

	Part I		Part II	
	Right	Wrong	Right	Wrong
Cases of Increase,	2	1	1	2
Cases of Decrease,	4	4	4	4
Equal Judges,	2	1	1	0

It will be noted that the increases are uniformly higher, in both Right and Wrong. The import of these figures should not be overrated, since most of the differences are not in themselves significant, yet their common trend would certainly show that we cannot assume, from the data here, the presence of anything like a regular or well-defined tendency. We may conclude, in short, that the table reveals no positive evidence of any significant tendency for either plus or minus errors to predominate; such as would normally follow from the action of a common, and consistent, causal factor.

On the basis of the evidence above cited, the writer believes we may with confidence accept that the particular form of procedure, here under consideration, introduced into our data no errors of consistent direction or significant size.

Three considerations, bearing on the problem, may here be referred to that may cast an influence not to be omitted. 1) As previously observed, the temperatures often rose or fell, between successive checks, by one or more hundredths; the intervals between them were thus correspondingly altered in the course of an afternoon. Hence, I-C may well cause that

no mere repetition of earlier judgments, however carefully made, could meet the situation; a fresh and careful scrutiny was constantly in order. This factor would naturally apply to series separated by an hour or so, rather than to repeated trials in the same series. 2) The constant errors involved in the space- and especially the time-orders must also be noted here. If in one order (2 preceding J) J seemed warmer, in the reverse order (J - 2) 2, instead of cooler, might also be rated warmer, depending upon the extent and direction of the constant error. Since the amount of this tendency was wholly unknown to E-O, while the direction could only be guessed at, judgment in opposed time-orders was necessarily independent. The same was true, in smaller degree, of reverse space-orders. 3) It must be noted, finally, that a careful and experienced O is less open to bias in a situation of this kind than might be expected. E-O by no means assumed a given jar to be warmer from the mere fact that he had so rated it once before. The preceding judgments, so far as they came to mind at all, served rather as a kind of hypothesis to be tested and scrutinized; his attitude toward them was essentially experimental, non-committal; they may have been correct, but a fresh examination might well turn out otherwise. The preceding judgments, furthermore, except when the objective interval was large, were rarely consistent among themselves. By the close of the practice period, O tends to take each pair of jars for itself, in objective and routine fashion; he comes to think less of preceding estimates and more in terms of the sense-impression; and when earlier trials do come to

mind, he can take them in a critical and impartial way. That this statement of a practiced observer's attitude is not fanciful, our analysis of the material above may serve to support.

The process of judging was carefully controlled. At the beginning of an hour-session, the four fingers of each hand were placed in the adaptation jar; the surface of the water was between the second and third joints, so that the fingers were well immersed, without wetting the rest of the hand. If the body of the hand inadvertently was wet, it was dried without delay, since the coolness was distracting. It was also found that if the fingers were wet beyond their point of junction with the hand, a disagreeably 'soggy' impression resulted that was not otherwise present. Both hands were thus retained in the adaptation jar, with the water freely circulating between the fingers, for a few minutes (three to five); the exact time depended upon the temperature and is given in each case with the results.

The question will inevitably emerge, and may as well be broached at this point: Was a period of three to five minutes adequate for proper adaptation? If we define the term, as does Thunberg:[1] "Wenn ein die Haut berührender Gegenstand,

[1]Thunberg, T.: Nagel's Handbuch der Physiologie des Menschen (1904); III. Bd., p. 670.

dessen Temperatur an der Berührungsfläche während der Berührung konstant bleibt, keine Empfindungen von Kälte oder Wärme hervorruft, ist die Hautstelle für diese Temperatur adaptiert"; if, in other words, we mean by adaptation complete thermal indif-

ference, complete absence of warm or cold sensation, then certainly this period is not, in general, adequate. No pretense is made that, by the end of five minutes, a temperature of, say, 16° gave no sensation of cold. If we define the term absolutely, as does Ebbinghaus, for example:[3] "die Abstumpfung

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[3]Ebbinghaus, H.: Grundzüge der Psychologie, I. Bd. (4 Aufl. 1919); p. 565.

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der Empfindungen bei kontinuierlicher Fortdauer der objektiven Reize", then, of course, the 'Abstumpfung' has advanced a certain way by the end of this interval; not so far, at extreme temperatures, as in a longer period. But at once the practical question arises: What degree of 'Abstumpfung' is demanded in order that judging may properly begin? That the 'Abstumpfung' proceed to complete indifference is, in general, not feasible, if indeed possible. The writer is much inclined to question - although not having specifically investigated this point, he is unable to speak with certainty - whether complete indifference is attainable beyond a limited range near skin temperature. According to Leonard,[4] the indifference temper-

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[4]Thunberg, T. op. cit., p. 671. Leonard cited by Thunberg.

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ature on the value side of the band could be depressed to almost 25° and raised to above 35° - a range of about 10°. Thunberg asserts,[5] however, that these values are "nach Reinec-

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[5]Thunberg, T: op. cit., p. 671.

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ung als Extreme anzusehen"; he found that "Man kann d.h. die

Finger für eine Temperatur von ungefähr 11° C. adaptieren, so dass man eine deutliche Kälteempfindung von einer konstanten Temperatur von 12° C. erhält, und man kann sie für eine Temperatur von 39° adaptieren, so dass nur eine unbedeutende Erniedrigung eine Kälteempfindung verursacht." The statement may be accepted as correct - indeed, our own data confirm it within the range 16° to 44°; but the facts by no means imply, as Thunberg seems to think, complete absence of thermal quality at the temperatures (11° and 39°) to which the fingers were 'adapted.' The writer wishes to cite three facts which summarize S-O's comments, as incorporated in the protocol with the original data.

1) Adaptation was not always 'complete' even at the close of a series, 45 minutes after it began. The fingers were not continuously immersed during this time, but were exposed to the air for brief intervals - about 5 seconds out of every 20, after the initial adaptation period - in the process of judging, as related below. This exposure, of course, was unavoidable, but limited as closely as possible.

2) Indifference, even if (partially or wholly) achieved, could not always be retained indefinitely; it was reported subject to noticeable, but usually unexplained, interruptions.

3) Throughout an entire series, the adaptation jar might have different thermal quality to the two hands: it might seem cooler to one or be indifferent to one and not to the other.

Consider the following illustrative comments:

25°. "Left hand again seems cooler in adaptation jar than right hand. Adaptation not complete by end

of hand - water still seems cool most of the time; at other times seems much warmer - coolness is irregular."

32°. "In this series to-day, left hand feels distinctly cooler in adaptation jar than right. This coolness much more conspicuous at some times than others."

27°. "Even after long-continued presence of hands in water, it still has temperature quality."

20°. "Water in adaptation jar felt indifferent or somewhat cool to left, but very cool to right, hand."

27°. "Late in hour, as well as earlier, 'waves' of coolness pass from finger-tips up forearm." (Not due to stirring, as shown by careful tests with dye-mass).

20°. "Sometimes a sudden change seems to occur; a jar in which the hands have been adapting for some time will suddenly feel noticeably cooler to one or both than before."

The interpretation in 2) rests by no means satisfactorily on fragment; they are noted, in the records, for the temperatures 20°, 25° and 32°; they appeared suddenly and, so far as determinable, with no change in any objective condition (e.g., temperature of water or air). They illustrate, however, that complete indifference, even if once achieved, cannot always be uniformly maintained, at least under the necessary conditions of our study.

1) and 3) above were much more typical and widespread. From the sensible attack, it will be seen that they

were by no means confined to extreme stimuli, but that they appeared with equal prominence in the vicinity of skin-temperature (22° and 32°).

It would seem obvious, in light of these facts, that a systematic program of investigation could hardly be carried through, were one to insist on complete indifference in both hands as a prerequisite to every judgment. At least four practical impediments to any such demand may be cited: 1) We should need an indefinite period for adaptation, not determinable in advance and apparently in excess of 45 minutes sometimes, as a preliminary to our first (or any) series; it is further strongly open to doubt whether indifference could be reached at all in some cases. 2) We should have to break off any series in case temperature-quality reappeared in either hand and continue our exposure to the adaptation-level until this departed, if at all. 3) If we keep the fingers continuously immersed for two hours or more, the skin becomes inflamed, wrinkled and coarse, attended with a pronounced grubbing and hardening of the normal sensitivity; so that judging would have to cease until normal skin-conditions could be restored.[1]

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[1] This was avoided in our procedure by means of an interval (15 minutes or so) between successive series, during which the hands were out of water.

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4) Adaptation is slightly disturbed in the very act of judging and would need to be reinduced, each time, to the point of subjective indifference. It surely needs no argument that work hedged within this complex of limitations would be almost intolerably hampered. The whole subject of adaptation, its

physiological basis and import, is far too obscure for us to set up an arbitrary standard, like complete indifference, as a prerequisite of accurate work in temperature-discrimination. Finally, our survey of the data, given above, revealed no appreciable effect, in direction or degree, upon the accuracy of discrimination in the course of a series; so that, if adaptation played any rôle, it left no material mark behind it or else its influence was neutralized by the operation of other factors.[1]

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[1]The reader is cautioned not to infer, from the above discussion, that adaptation was grossly inadequate and variable in our work; far from it. By the time judging began, adaptation was regularly well advanced, and only a few times was the difference in sensation between right and left hands at all conspicuous. A good approach to indifference can be secured with relative ease; but complete adaptation is quite another matter.

It was therefore decided to begin judging after a brief, and for any temperature uniform, period;[2] so that the status

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[2]The shortest period (3 minutes) was used near skin temperatures; the longer ones (4 and 5 minutes) at higher and lower levels.

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of adaptation remained, for each series, relatively constant.

At the close of the adaptation period, the recorder read the first combination to be compared. E-O's hands moved in time with a metronome beating seconds. At a given stroke, the hands were lifted from the adaptation jar, remaining in the air just two beats (seconds); on the third, they were immersed in the first jar to be compared, remaining there two seconds; removed at the fifth beat, again moving in air two seconds; finally, in second stimulus-jar for two beats, with-

drawn at the ninth stroke and returned directly to the adaptation jar which was also the standard, in each case. The judgment was given when the hands were leaving the second jar; the discrimination was regularly made in terms of the second jar, but spoken and recorded in terms of J.[1] The above procedure

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[1]Assume J to be warmer than S; in the order S-J, the judgment is made and spoken in terms of the second, which is also J; in the order J-S, the judgment is mentally (or 'subvocally') formulated in terms of the second jar, S, which is cooler; but is spoken and recorded as W, applying to J. The writer wishes to emphasize that he could observe no essential difference in the movement of attention, in these two orders; in J-S the judgment formed in terms of S was simply reversed for the convenience of greater uniformity in the records. With a little practice this became easy and habitual, having, so far as could be determined by K-O, no distracting effect.

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was correspondingly modified to accommodate, for example, the method of equivalents, in which right and left hands are simultaneously immersed in the two comparison jars; but in all methods the time of exposure to each stimulus was two seconds, at all temperatures, and exposure to the air, preceding either stimulus, was likewise fixed at two seconds. The need for this regulation is apparent. The temperature of the hands, when exposed to air, either rises or falls (usually the latter). It is necessary, not only that the amount of this change be the same in each hand, but also that it remain constant from one judgment to the next. This can only be achieved, approximately, by a fixed time of exposure. The change in temperature, owing to passage through the air, is an unavoidable feature of the method and introduced a constant error into our data; along with the results will also be given the means

used to eliminate this error. It could, in theory, be prevented by so regulating the room-temperature as to neutralize the warming or cooling effect; which, of course, in ordinary conditions, cannot be done. In this particular, a device like Thunberg's Temperator would seem better for investigating our problem, though apparently a good deal less feasible in other ways. In the later experiments (Group II), the air-temperature was measured with a thermometer suspended directly above the apparatus and recorded for each afternoon.

After the initial period of adaptation, for example, to jar 1, the five combinations of that jar with the remaining constants (1-2, 1-3, 1-4, 1-5, 1-6) were each compared four times, as above explained; number 1 being both standard and adaptation jar throughout. The hands were then transferred to the second jar and retained there one minute for adaptation, whereupon the five combinations of number 2 with the other jars were also compared four times. The hands were then removed, once more, to number 3; and so on, through the whole set of six jars. It will thus be seen that the adaptation jar was also the standard, by this method; and that the adaptation-temperature varied, from one jar to the next, by the amount of their difference. Inasmuch as this difference rarely exceeded 0.25°, it may be considered negligible beside the interval of 4° between the temperature levels investigated.

Just twenty seconds elapsed from the beginning of one comparison to that of the next; so that 120 judgments occupied 40 minutes, apart from the adaptation time. The compar-

lasa occupied from 6 to 9 seconds; 11 to 12 seconds were thus available for interval adaptation, before the next exposition began.

Some pairs of jars being nearer together than others, the hands had to move farther in comparing, say, 1-3 than 1-2. This more extensive range would, of course, have greater inertian effect, even though the time of exposure to the six was alike in both cases. To obviate this, the hands were kept moving, while in the air, at about uniform rate for the whole two seconds, whether the distance between the jars was large or small.

An interval of two seconds was chosen because: 1) from experiments on attention, 1 - 1/2 to 2 seconds is commonly accepted as the optimal time for passing to sense-impressions, that is, for adjusting the attention to a given stimulus; it seemed therefore the best interval for exposing the receptors to the two perception-stimuli; 2) it was long enough to enable S-O to pass from one jar to another without unpleasant end, at the same time, without keeping the hands in the air longer than necessary.

Care was used always to immerse the hands to the same depth when making comparisons, because, as well known, stimulation of a larger area often gives the illusion of greater intensity of stimulus. There is here the additional factor that if the hand be incompletely stimulated above the level of adaptation, the unadapted area often yields a distinctly stronger warm or cool sensation than the adapted part, and thereby judgment is confused. With practise the depth of immersion

was kept uniform and consistent; and while, in a few individual cases, the judgment may well have been distorted by too deep immersion, our method of statistical treatment presupposes variable errors of this type and is designed to care for them.

CHAPTER IV

STATISTICAL TREATMENT OF DATA

The data were treated according to Urban's modification of the psychometric function, Phi of gamma. The essential features of this method will here be briefly given.[1]

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1. An extended presentation may be found in the article by F.M.Urban: Die psychophysischen Massmethoden als Grundlagen empirischer Messungen; Archiv fur die gesamte Psychologie, Vol. XV (1909), pp. 261-355, and XVI, pp.168-227 (especially pp.177-227 of the latter volume). A critical presentation appears in W. Wirth: Psychophysik, and in Brown and Thomson's Essentials of Mental Measurement. An elementary summary is given by E. G. Boring in American Journal of Psychology, XXVIII (1917), pp.280-293.

Let s and j be two stimuli, a standard and a comparison stimulus, presented to an O under specified conditions; let O be instructed to respond to j, upon each presentation, in one of n ways (for example, with one of the judgments Warmer, Doubtful, Cooler); let p_w be the percentage of W judgments in the total number of judgments given at the point j. This method now makes two assumptions: 1) that $p_w = f_w(j)$, that is, that p_w is some (determinable) function of the corresponding value of j; and 2) that this functional relationship may be expressed by the fraction

$$p_w = \frac{\int_{-\infty}^{j} e^{-h_w^2 (x - a_w)^2} dx}{\int_{-\infty}^{+\infty} e^{-h_w^2 (x - a_w)^2} dx} \qquad (4)$$

The denominator here equals $\frac{\sqrt{\pi}}{h}$ (for proof, see books on Least Squares, such as, Weld: Theory of Errors and Least Squares, p.180 f.; cf. Wilson: Advanced Calculus, p.364). The equation thus becomes

$$p_w = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{j} e^{-h_w^2 (x - a_w)^2} h_w \, dx \qquad (5)$$

Place $z = h(x-a)$. Since x ranges from $-\infty$ to j, z will range from $-\infty$ to $h(j-a)$. We then have

$$p_w = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{h_w (j - a_w)} e^{-z^2} \, dz \qquad (6)$$

When $j = a$, $h(j-a) = 0$, and $f(j) = 1/2$ (since the area of the integral from $-\infty$ to $+\infty$ equals unity). In this equation, j has already been defined as the comparison-stimulus, to which the judgments as given apply; a may now be defined, therefore, as that value of j for which $f(j)$ equals 1/2; in other words, a is the abscissal value which divides the area represented by the integral into two equal parts, hence the median. This median point is defined, in the present method, as the limen, the principal value which the equation yields. h is commonly known as the 'measure of precision'; the greater its value, the smaller the variability of the normal distribution whose area is represented by the integral above given

$$A_w = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-h_w^2 (x - a_w)^2} dx \qquad (7)$$

In terms of the standard deviation, a value more commonly used ineducational statistics, $\underline{h} = \frac{1}{\sigma\sqrt{2}}$. Since $\underline{a}$ equals the median (that is, the origin of the Gaussian differential curve corresponding to our integral) and $\underline{j}$ is the variable, $\underline{j-a}$ equals the difference, usually called $\underline{x}$. $\underline{hx} = \frac{x}{\sigma\sqrt{2}} = \gamma$. Hence our equation reduces to

$$p_w = \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\gamma_w} e^{-z^2} dz = \frac{1}{2} + \frac{1}{2} \phi(\gamma_w). \quad \left[\phi(\gamma) = \frac{1}{\sqrt{\pi}} \int_{-\gamma}^{+\gamma} e^{-z^2} dz \right] \qquad (8)$$

This being the final reduction, the method is usually known as the 'Phi of Gamma' method.

Special tables giving the values of gamma with the corresponding values, Phi of gamma - that is, $2(p - 1/2)$ - are found in certain works on the theory of probabilities (e.g., H. Bruns: Wahrscheinlichkeits-rechnung, Anhang, (1906), contains values of gamma from .000 to 2.89). The value of gamma can also be determined from the customary tables of the probability integral (such as, Sheppard's tables in K. Pearson: Tables for Statisticians and Biometricians, p.2.), in which the value of $\frac{x}{\sigma}$ corresponding to any given area, $\frac{1}{2} + \frac{1}{2}\phi\left(\frac{x}{\sigma}\right)$, is given. Since gamma equals $\frac{x}{\sigma\sqrt{2}}$, its

value can be readily secured. For example, when gamma equals 1, Phi (gamma) = .8427 (from Bruns' table), or 1/2 Phi (gamma)= .4213. From Sheppard's table, when 1/2 Phi ($\frac{x}{\sigma}$) = .4213, $\frac{x}{\sigma}$ = 1.414, which equals, as the above relation indicates, $\gamma\sqrt{2}$.

Our equation thus contains two unknowns, h and a; a being the median-point, here defined as the limen, and h being the precision of the distribution, from which the s.d. or p.e. of the limen may be determined. Given two values of gamma, γ_{w_1} and γ_{w_2}, derived from two values of p_w, p_{w_1} and p_{w_2}, for two values of j, j_1 and j_2, it is obvious that h_w and a_w would be uniquely determined from the two equations:

$$\gamma_{w_1} = h_w (j_1 - a_w)$$

$$\gamma_{w_2} = h_w (j_2 - a_w) \qquad (9)$$

Since, however, it is desirable to have determinations of p_w for more than two values of j, we must calculate the most probable value of h and a by the usual method of least squares.

For this purpose the equations must be in linear form. The relation between p, the observed value, and the unknowns, h and a, is however far from linear, involving the transcendental function Phi (gamma). The reduction to linear form may be directly accomplished in the following way (cf. Wirth, pp.136 ff. and 205 ff.); Approximate values for h and a may be found by solving any two of the observation equations,

of simple combinations of them (ibidem, p.205). Let these approximate values be $\underline{h}'$ and $\underline{a}'$; $\underline{h}'$ may be treated as equal to the most probable value, $\underline{h}$, plus some correction (presumably small), $\underline{d}$; likewise $\underline{a}'$ equals $\underline{a}$ plus some correction $\underline{e}$. Substituting these values, we have

$$p = \tfrac{1}{2} + \tfrac{1}{2}\,\phi(y) = \tfrac{1}{2} + \tfrac{1}{2}\,\phi\left[h(j-a)\right] = \tfrac{1}{2} + \tfrac{1}{2}\,\phi\left[(h'-d)(j-a'+e)\right] \tag{10}$$

Expanding the final form into a Taylor series in powers of $\underline{d}$ and $\underline{e}$, we get

$$\phi\left[(h'-d)(j-a'+e)\right] = \phi\left[h'(j-a')\right] - d\,\frac{\partial\phi}{\partial h'} - e\,\frac{\partial\phi}{\partial a'} + \frac{1}{2!}\left(d^2\frac{\partial^2\phi}{\partial h'^2} + 2de\,\frac{\partial^2\phi}{\partial h'\,\partial a'} + e^2\frac{\partial^2\phi}{\partial a'^2}\right) + \cdots \tag{11}$$

Only the first power of $\underline{d}$ and $\underline{e}$ can be used, since the equation is to be linear. The approximation thus secured will, however, be adequate in case $\underline{d}$ and $\underline{e}$ are small; otherwise the process must be repeated.

The above 'Korrektionsverfahren' of Müller and Fechner is, however, less convenient in practise than the 'Gewichtsverfahren' first developed by Müller. As before, we start with a series of equations

$$\text{gamma}_k = \underline{h}(\underline{j}_k - \underline{a}) \qquad (\underline{k} = 1,2, \text{-----}, \underline{n}) \tag{12}$$

We must now secure the most probable values of $\underline{h}$ and $\underline{a}$ on the

basis of the actually observed quantities, p_k, so that the mean-square deviation of the observed from the calculated p's will be a minimum. That is, the mean-square adjustment can properly be applied only to the directly observed quantities, p, not to derived quantities such as gamma. In order to use our series of linear equations in gamma, we must therefore determine a weight, Psi, such that

$$Psi_k(gamma_k - gamma_k^\circ)^2 = (p_k - p_k^\circ)^2 \quad (k = 1, 2^\circ ---, n) \quad (13)$$

p_k° here denotes the 'most probable' value of p_k, that is, the adjusted value of p_k corresponding to h° and a°, the most probable values of h and a; while $gamma_k^\circ$ is the value of $gamma_k$ directly calculated from p_k°. The weight, Psi, must be such that

$$\sum_{\kappa=1}^{n} \psi_\kappa (\gamma_\kappa - \gamma_\kappa^\circ)^2 = \sum \psi_\kappa t_\kappa^{\circ 2} = \sum (p_\kappa - p_\kappa^\circ)^2 = a\ minimum. \quad (14)$$

Let $(gamma_k - gamma_k^\circ) = t_k^\circ$. The 'most probable' calculated value of p_k is $p_k^\circ = 1/2 + 1/2\ Phi\ (gamma_k^\circ)$; (15)

the observed value is

$$p_k = 1/2 + 1/2\ Phi\ (gamma_k); \quad (16)$$

hence,

$$(p_k^\circ - p_k) = 1/2\ Phi\ (gamma_k^\circ) - 1/2\ Phi\ (gamma_k)$$
$$= 1/2\ Phi(\ gamma_k - t_k^\circ) - 1/2\ Phi(gamma_k). \quad (17)$$

Expanding $1/2\ Phi(gamma_k - t_k^\circ)$ into a Taylor series in powers

of $\underline{t}^{\circ}$, we have $1/2 \left[\text{Phi}(\text{gamma}_k) - \underline{t}^{\circ}_k \{\text{Phi}'(\text{gamma}_k) + (1/2!)\underline{t}^{\circ 2}_k \text{Phi}''(\text{gamma}_k) - (1/3!)\underline{t}^{\circ 3}_k \text{Phi}'''(\text{gamma}_k) + \cdots \right]$ (18)

Deducting $1/2\ \text{Phi}(\text{gamma}_k)$ and performing the differentiations indicated, we get

$$(p^{\circ}_k - p_k) = -\frac{t^{\circ}_k}{\sqrt{\pi}} e^{-\gamma^2_k} - \frac{t^{\circ 2}_k \gamma_k}{\sqrt{\pi}} e^{-\gamma^2_k} + \frac{t^{\circ 3}_k}{3\sqrt{\pi}} e^{-\gamma^2_k}(1 - 2\gamma^2_k) \tag{19}$$

Obviously, the larger the number of terms in the right-hand member, the nearer will its value approach that of $(\underline{p}^{\circ}_k - \underline{p}_k)$. By employing the Phi(gamma)method, however, we have already assumed that the series, $\underline{p}_1, \underline{p}_2, \cdots, \underline{p}_n$, forms an approximately Gaussian ogive or integral curve. If this postulate is valid, the value of $(\underline{p}^{\circ}_k - \underline{p}_k)$ is small, and likewise $\underline{t}^{\circ}_k$ is small enough that its high powers may be neglected in the above series. Müller and Urban, in fact, retain only the first power of $\underline{t}^{\circ}_k$, so that

$$(\underline{p}^{\circ}_k - \underline{p}_k)^2 = (1/\pi)\underline{t}^{\circ 2}_k e^{-2\gamma^2_k} \tag{20}$$

and it is on this basis that their tabulated weights are derived. This procedure is accurate enough when the deviations are small $(\underline{t}^{\circ}_k)$; but that they are otherwise not satisfactory may be shown by an example taken from Urban's own work.

Take the results for observer I, heavier (grösser) judgments, as given in the Archiv, XV, p.287. The values of $\underline{p}_k$ for the comparison-stimuli, 84, 88, ---, 108, range from 0.0022 to 09400. The most probable values of $\underline{h}$ and $\underline{m}$, cal-

culated from these p's, are given in XVI, p.192, Tabelle 44, as 0.1361 and 99.68. The actual values of gamma, and the corresponding weights P_k for observer I are given in XVI, p.190, Tabelle 40. If the values of gamma_k° are calculated from $h^\circ(j_k - a^\circ)$ and deducted from the actual values, very nearly the following residuals will be obtained, in order: .1190, .1376, .0924, -.0400, -.1985, .3008, -.0330. The sum of these, when squared and multiplied by the weights, P, equals .098450. (Urban's value for this sum is .098550; see XVI, p. 226, Tabelle 68). If we now calculate the values of p_k° by inserting the most probable values of h and a into the equation

$$p_k^\circ = \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{.1361\,(j_k - 99.68)} e^{-x^2} dx \qquad (21)$$

and deduct them from the actual values, p_k, the following residuals will be obtained in order: .0009, .0077, 0192, -.0172, -.1113, .0984, -.0054. The sum of these, when squared and multiplied by the corresponding value of $1/pq$ (cf. below, this paragraph), equals .164821. This gives us the left member of equation (20) above; to secure the right member, we proceed as follows: The 'true' Urban weights are equal to

$$\frac{1}{\pi} \cdot \frac{1}{p_k q_k} e^{-2\gamma_k^2} \qquad (22)$$

In tabulating, however, each weight is multiplied by $\frac{\pi}{4}$; this makes a more convenient table, since the maximal value is then unity instead of an improper fraction. (This is of

course permissible, since the relative value of the weights is unchanged; and it is their relative value alone that is of importance in solving the normal equations). If equation (20) is approximately true, then

$$\frac{1}{p_k q_k}\left(p_k^\circ - p_k\right)^2 = \frac{1}{\pi}\cdot\frac{1}{p_k q_k}\cdot e^{-2\gamma_k^2}\cdot t_k^{\circ 2} \qquad (23)$$

approximately. The left-hand member of this equation we have above found to be 0.164821. The right-hand member equals $\frac{4}{\pi}$ times 0.098450 = 0.1254. This is a distinctly poor agreement with 0.1648, and shows that the above equation can hardly be called satisfactory. If, however, the second-power term of Taylor's series (19) is retained in the weight, we have

$$\left(p_k^\circ - p_k\right)^2 = \frac{1}{\pi}\cdot t_k^{\circ 2}\cdot e^{-2\gamma_k^2}\left(1 + t_k^\circ \gamma_k\right)^2 \qquad (24)$$

If the right-hand member in (23) is calculated from this weight we get the value 0.1643, which, considering the numerous interpolations and approximations necessary, may be called a good agreement with 0.1648.

The imperfect approximation of the tabulated weights can not be remedied, however, in the tables, since each succeeding term in the series involves t_k° as an integral part of the weight. In spite of their approximate character, the

tabulated weights have been used in this study.

The Urban weight contains an additional factor, $1/pq$, which differentiates it from the original Muller weight. It is assumed that the percentage, p_k, which we obtain at any given point is itself merely a sample from a normal distribution; that is, if we took an indefinite number of sets of judgments at the same point, (say, 1000 sets of 100 judgments each when $j = 32.10°$ and $s = 32.00°$) that the 1000 values of p_k derived from these sets would form a closely normal distribution about their mean value. In other words, the sequence of percentages obtained

$$p_{k_1}, p_{k_2}, \cdots, p_{k_n},$$

is assumed to be a true Bernoullian series, the factors conditioning the appearance and non-appearance of the given judgment remaining constant through all the series. (cf. Arne Fisher: Mathematical Theory of Probabilities; pp.118 ff.). The standard deviation of this distribution equals $\sqrt{\frac{pq}{n}}$ ($q = 1 - p$). (For proof, cf. ibidem, p.119 and 110). If n (the number of samples of p_k obtained, in our example 1000) remains constant, it is evident that the standard deviation of these distributions obtained at different points, j_k, is not constant, since pq varies. The product pq is a maximum when $p = q = 1/2$ and decreases to zero, when p or q equals zero. In the process of fitting the most probable

ogive to these percentages, each P_k is weighted, in the customary manner, inversely proportional tothe square of sigma or directly proportional to h^2. The weight, therefore, of any p_k is $n/p_k q_k$; this value is minimal at $p = 1/2$ and becomes infinite when $p = 0$ or 1. Since the factor, $e^{-2\gamma^2}$, however, decreases more rapidly than n/pq increases, the total weight is maximal at $p = 1/2$ and decreases to zero at $p = 0$ or 1 (cf. Urban, XVI, p.184).

The combination of these two weights, the Bernoullian and Muller's, gives us the final Urban weight

$$P = \frac{n}{\pi} \cdot \frac{1}{pq} e^{-2\gamma^2} \qquad (25)$$

which, as above stated, is tabulated in the more convenient form,

$$\frac{1}{4pq} e^{-2\gamma^2}.$$

If the number of judgments n secured at each point, I_k, in the series is constant, the factor n may be omitted in the above weight; when n varies from one set to another, as is the case in this study, it must be included in the weight.

We are now ready to secure the normal equations for h and a from the n actual equations. Given,

$$\begin{aligned} \text{gamma}_1 &= h(i_1 - a), \text{ with the weight } P_1, \\ \text{gamma}_2 &= h(i_2 - a), \quad " \quad " \quad P_2, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \text{gamma}_n &= h(i_n - a), \quad " \quad " \quad P_n. \end{aligned} \qquad (26)$$

Multiplying each equation by its weight and by the coefficient of $\underline{h}$, and summing, we have for the normal equation in $\underline{h}$,

$$[Pj\gamma] = [Pjj]\underline{h} - [Pj]\underline{ah}. \tag{27}$$

The equation in (ah) is

$$-[P\gamma] = -[Pj]\underline{h} + [P]\underline{ah}. \tag{28}$$

Solving simultaneously, we have

$$\underline{h} = \frac{[P][Pj\gamma] - [P\gamma][Pj]}{[P][Pjj] - [Pj][Pj]} \tag{29}$$

$$\underline{a} = \frac{[Pj]\underline{h} - [P\gamma]}{[P]\underline{h}} \tag{30}$$

It remains to determine the probable errors of h and g. Urban, in his study, computed the p.e.'s of the upper and lower limens, as well as of the 'intervals of uncertainty' - the interval between upper and lower limen - and obtained values which, in every case, were larger than the quantity to which it applied; they ranged from 1½ to 10 times the size of the limens, or intervals of uncertainty, affected. (For Urban's formula and results, see XVI, 285 ff. and 192). These discouraging figures lead him to conclude his monograph with the statement: "Die sogenannte Konstanzmethode ist demnach die ungenaueste der hier dargelegten vier psychophysischen Methoden" (p. 287). That his p.e.'s cannot be correct would appear from several considerations:

1) Viewed in the light of general considerations only, it would seem strange that results, secured under the best conditions from observers as skilled as Professor Urban himself,

should have p.e.'s so large as almost to deprive them of value. Urban ascribes it to the method, whose inaccuracy is due to the fact that "nicht nur die Fehler der Beobachtungen über die Wahrscheinlichkeiten der zugelassenen Urteilsarten, sondern auch die Fehler der Theorie in das Resultat eingehen" (p. 227). This seems like a forced explanation, however, since 2) he shows (p.224 f.) that the Phi (gamma) hypothesis agrees better with the observed data than does the arc tan hypothesis, the other psychometric function that he tested; this would hardly justify the statement that the Phi(gamma) method is the least accurate of all. 3) In the third place, as Thomson has pointed out,[1] since the 450

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1. Thomson, G. H.: Accuracy of the Phi(gamma) Process; British Journal of Psychology, VII, (1914), p.53.

---

judgments were taken in 9 separate groups of 50 each, and h and a were obtained for each group separately, we can get an actual p.e. from these nine values which should roughly agree with the theoretical one. The p.e. of a, computed in this way directly from the nine values of a, equals .276, which is of an entirely different order of magnitude than Urban's value, 9.477.

Thomson, in the paper above cited, showed by other means that Urban's values are incorrect; and has developed a formula for computing the p.e. out of the customary basic formula:

If Z is any function, f, of z_1, z_2, ---, and if X is the error produced in Z by the concurrence of errors x_1 in z_1, x_2 in z_2, ---, then

$$Z + X = f\ (z_1 + x_1)(z_2 + x_2) \text{ ---} \qquad (31)$$

Expanding the second term into a Taylor series in powers of x_1, x_2, ---, and deducting $Z = f(z_1, z_2, \text{---},)$, we have

$$X = x_1 \frac{\partial f}{\partial z_1} + x_2 \frac{\partial f}{\partial z_2} + \cdots + \frac{1}{2!}\left(x_1^2 \frac{\partial^2 f}{\partial z_1^2} + 2 x_1 x_2 \frac{\partial^2 f}{\partial z_1 \partial z_2} + x_2^2 \frac{\partial^2 f}{\partial z_2^2}\right) + \cdots \qquad (32)$$

Assuming that the deviations are small enough to neglect all powers of x_1, x_2, ---, beyond the first, we have the usual formula

$$X = x_1 \frac{\partial f}{\partial z_1} + x_2 \frac{\partial f}{\partial z_2} + \cdots \qquad (33)$$

from which

$$\text{p.e.}_Z^2 = \left(\frac{\partial f}{\partial z_1}\right)^2 (\text{p.e.}_{z_1})^2 + \left(\frac{\partial f}{\partial z_2}\right)^2 (\text{p.e.}_{z_2})^2 + \cdots$$

(cf. Merriman: Method of Least Squares; pp.78 f.) (34)

This formula is an acceptable approximation only when the p.e.'s of r_1, r_2, ---, are quite small. The p.e._p is here obtained by the Bernoullian formula

$$\underline{p.e.}_p = .6745\sqrt{\frac{pq}{n}} \qquad (\underline{n} = 450, \text{ here}) \qquad (35)$$

so that the maximal value of $\underline{p.e.}_p$, in this case, is about .016.

It is possible, however, to derive the p.e. of both h and a directly from h itself. This quantity is the 'measure of precision' of the normal curve corresponding to the Gaussian integral which is fitted to the observed percentages. The s.d. of this distribution is $\frac{1}{h\sqrt{2}}$ and the p.e. of the median, m, therefore, = .84535 $\frac{\sigma}{\sqrt{n}}$ = .84535 $\frac{1}{h\sqrt{2n}}$. Since, by the customary formulas, $\underline{s.d.}_\sigma = \frac{\sigma}{\sqrt{2n}}$, the s.d. of h = $\frac{h}{\sqrt{2n}}$. Applying these equations directly to the Urban values discussed by Thomson, we have h = .1361; n = 450; $\underline{s.d.}_h$ = .0045 and $\underline{p.e.}_h$ = .0030 (Thomson's value = .003). The s.d. of the distribution = 5.1955, whence the p.e. of a by the above formula = .207 (Thomson's value = .151).

The difference between these last two values is considerable. We have now to choose between these two methods: the Thomson method aims to derive the p.e. from the actual values of p, the h-method gives the p.e., for the Gaussian integral fitted to the percentages. Neither value is exact; Thomson's, for two reasons: 1) because of the abbreviated p.e. - formula which he uses; and 2), what is probably more important, because of the merely approximate values of P

(the Urban weights), which appear very frequently in his formula. We saw above how inexact the Urban weights are in this particular case. The value derived from h is not exact, on the other hand, for the reason that it gives the p.e. of the smooth curve instead of the observed values. Neither method gives a strictly reliable p.e. except when the deviations of the observed from the adjusted p's are small; which is more exact will depend upon the individual case. In any event, the Thomson method seems to have no outstanding advantage to compensate the extremely laborious computation it involves. The h-method has therefore been used throughout in this study. It may be noted that the s.d. calculated from h is confirmed by Wirth's interpolation-method of computing s.d. (cf. p. 192, towhich the reader is referred for the original derivation). The Wirthian formula requires that the extreme value, where p = 1.00, be known. Since this point is not given in Urban's series, the highest p being .94, we cannot determine s.d. exactly by Wirth's method; but we may assume with practical certainty that, there being 94% heavier judgments at 108 grams, the percentage would rise to 100 at either 112 or 116 grams, or at least so near to 100 that the error would be negligible. If 100% occurs at 112, the s.d., by Wirth's formula, = 5.141; if at

116, s. d. = 5.220= it will be observed that our value (5.1955), derived from h, lies between the two.

Since the p.e. of a varies inversely with the square-root of its weight, n, that is, the area of the Gaussian curve fitted to the observed p's, the question remains: What is the value of n when the number of judgments upon which the different p's are based is not the same? When each p represents a certain percentage of the same number of judgments (450 in the example from Urban used above), then the total area of the normal curve (n) obviously is this constant number (450). When, however, the p's are not based on the same number of judgments, as in our study, what must we assume for the area (n) of the fitted curve? It was found by this formula:

$$\frac{\sum P_K n_K}{\sum P_K} = n_m \qquad (K = 1, 2, \cdots, r) \qquad (36)$$

where n_m is the weighted mean value of n. As an illustration of the procedure, consider the following series of W judgments where the standard temperature equals 20.00°:

J	19.85°	19.90°	19.95°	20.00°	20.05°	20.10°	20.15°
p	.09	.29	.44	.51	.69	.69	.90
n	11	89	107	85	110	59	20
P	.51	.89	.992	1.00	.91	.91	.54

(J is the comparison-temperature to which the judgment, as given, applies; p is the percentage of Warmer judgments; n is the total number of judgments taken at that level; P is the Urban weight).

The sum of $P_k n_k$ here = 440.55; the sum of P_k = 5.752; therefore, n_m = 76. 59. In other words, if 76.59 judgments had been taken in each of the seven series above, the p's remaining what they now are, our results would have been the same as they now are.

CHAPTER V

PRESENTATION AND DISCUSSION OF RESULTS

The numerical data summarized from the original records are reproduced in full in the tables in the Appendix. The results for K-O are tabulated under four different methods all involving constant stimulus-differences; the data from other observers were secured by various forms of the method of limits. The principal facts and conclusions relating to these groups will here be exhibited and correlated.

Constant Method A

These data, constituting a part of the writer's observations, are the most complete and extensive in our study. The procedure here reached its definitive form and these results are given first place in our conclusions. The details of procedure have been given in full, in our presentation of the method of securing data; hence will not be repeated here. In method A, both hands were successively exposed to standard and comparison jar; each constant served, in turn, as both adaptation-jar and standard, the others being then the comparison-jars.

Let the reader now consider Table 13 and Figure 5, in which the results are summarized in graphical form. The limens as there shown are not the values which emerge directly from the original calculations, but are 'corrected'.

TABLE 13

CONSTANT METHOD A.

Mean Values of L_w and L_c(corrected) for each Temperature

	28°	24°	20°	16°	20°	24°	28°	44°	40°	36°	32°	28°
L_w	.076	.064	.048	.067	.059	.039	.046	.078	.081	.062	.041	.038
P.E. L_w	.013	.013	.010	.012	.010	.006	.015		.040	.008	.006	.006
L_c	.088	.056	.061	.076	.056	.042	.046	.097	.075	.055	.038	.033
P.E. L_c	.015	.011	.013	.013	.010	.007	.015		.055	.008	.006	.005
$I(=L_w-L_c)$	.164	.120	.109	.143	.115	.081	.092	.175	.156	.117	.079	.071
P.E. I	.020	.017	.016	.017	.014	.009	.022		.068	.012	.008	.008

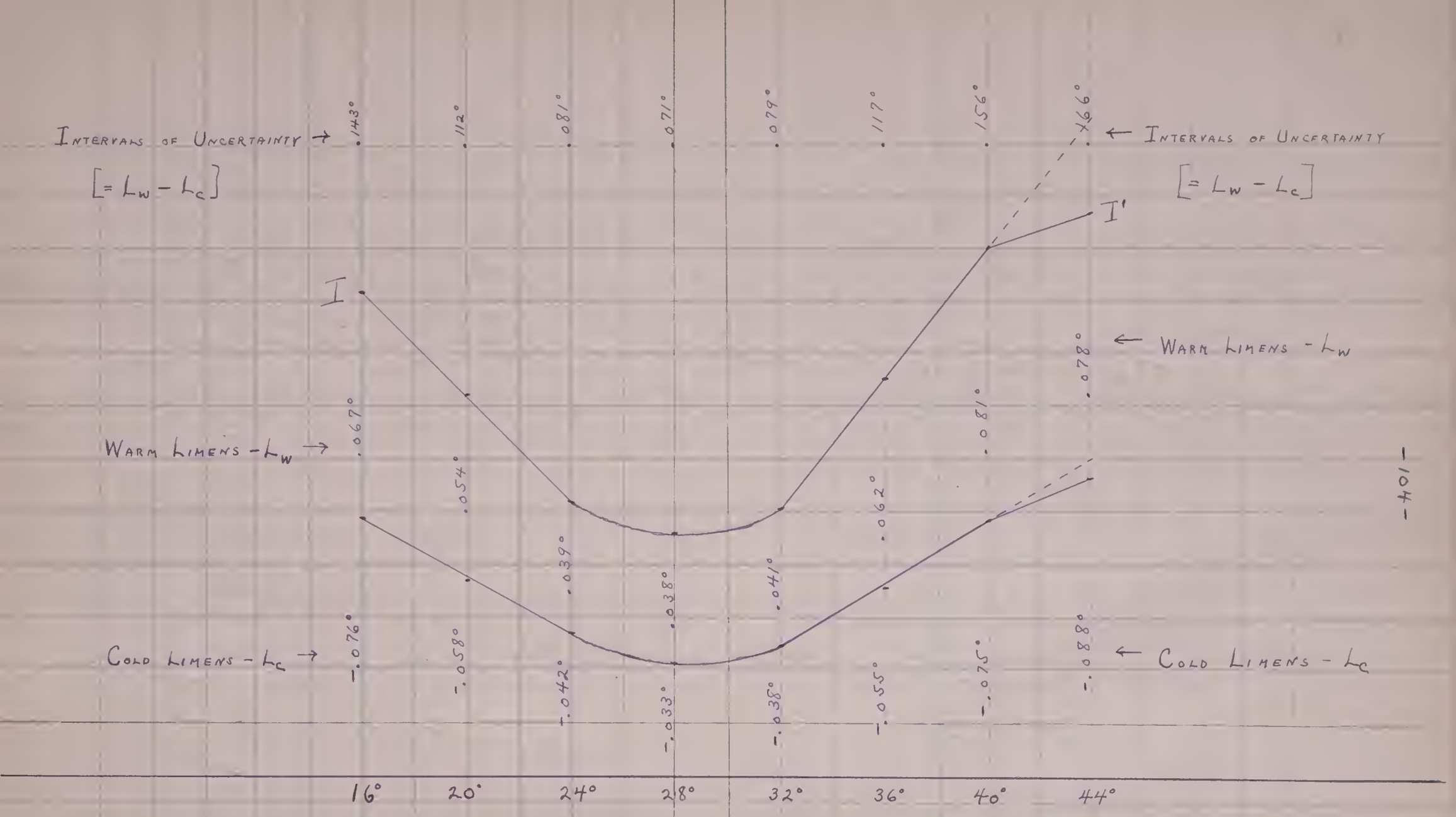

Fig. 5 - Constant Method A.

Adjustment Equations for II':

16°-24° $y = .1426 - .0308x$ (x = o at 16°).
24°-32° $y = .081 - .019x + .009a^2$ (x = o at 24°).
32°-40° $y = .0790 + .384x$ (x = o at 32°).

Minimal Value: y = .071 at 28.22°.

Observed and Adjusted Values for 44° doubtful. Values are identical, in every case, to third decimal.

Equations for Warm Limens:

16° - 24° $y = .0671° - .0139°x$ (x = o at 16°).
24° - 32° $y = .0391° - .0025°x + .0017°x^2$ (x = o at 24°).
32° - 40° $y = .0414° + .0198°x$ (x = o at 32°).
44° Doubtful
32° - 44°: →
Minimal Value: y = .038° at 26.9°.

Agreement of Observed and Adjusted Values:

	16°	20°	24°	32°	36°	40°
Observed	.067°	.054°	.039°	.041°	.062°	.081°
Adjusted	.067°	.053°	.039°	.041°	.061°	.081°

16°-24° Slope of Adjustment Line= .0139° per interval of 4°; whence Weber Constant = .0035

32°-40° Slope of Adjustment = .0198° per interval of 4°; whence Weber Constant = .005

Equations for Cold Limens:

$y = .0755° - .0169°x$ (x = o at 16°).
$y = .0418° - .0161°x + .0072°x^2$ (x = o at 24°)
$y = .0378° + .0186°x$ (x = o at 32°)
$y = .0387° + .0169°x.$ (x = o at 32°).

Minimal Value: y = .033° at 28.5°

Agreement of Observed and Adjusted Values:

	16°	20°	24°	32°	36°	40°
	.076°	.058°	.042°	.038°	.055°	.075° .088
	.076°	.059°	.042°	.038°	.056°	.075° "
			.039°	.056°	.073°	.088°

Slope of Adjustment Curve = .0169° per interval of 4°; whence Weber Constant = .004

Slope of Adjustment = .0186° per interval of 4°; whence Weber Constant = .0045

Any limen, when computed, is affected with a combination of constant tendencies: time-, space-, and other errors. It might be thought that, by combining the limens for the four time-space orders, we could get rid of these errors; and while this is approximately true, other tendencies still remain, imbedded in the limens, as will be seen later, which can not thus be eliminated. We therefore determine in each case the point Xi (cf. Notes toTables, below, p.157), at which a W and a C judgment are equally probable and use this as the zero- or indifference-point from which the true limen is reckoned. By the 'corrected' Warm limen, therefore, we mean L_w - Xi; that is, the distance from the point Xi, where $p_w = p_c$, to the median point of the Warmer curve, where $p_w = .50$. The true colder limen, likewise, is L_c - Xi.

The use of Xi as the 'dividing-point' between W and C is founded on these considerations: Our method of statistical treatment assumes that the responses 'W' and 'C' turn up as do Black and White balls drawn from an urn under specified conditions; that is, that each judgment is produced by the concurrence of an indefinite, and very large, number of factors, each of which is equally likely to be present or absent; or, in other words, that the judgment is the algebraic sum of such a set of factors, each acting, now positively now negatively, with equal likelihood. Where this is true, as well known,

the percentage of white balls drawn in, say, 1000 sets of 100 each, will closely approximate a 'normal' distribution. The zero-point of such a curve is taken as the point where the algebraic sum of these tendencies is zero, where the factors yielding 'white' and those yielding 'black' are exactly balanced. By a natural analogy, the zero- or indifference-point between W and C is where the W and C tendencies are equally strong, so that their algebraic sum equals zero. In the tables the position of Xi, and the corrected limens, are always given.

It will be noted that, in Group I, we started at 32°, passing down by 4° intervals to 16° and then coming back to 28°. We can thus determine, by comparing the first series with later ones at the same temperature, at what point practise effects are no longer discernible. Consider the following table:

	Beginning			Later		
	W	C	I	W.	C	I
32° (S-J only)	.080	-.100	.180	.031	-.038	.069
28°	.076	-.088	.164	(I) .046	-.046	.092
				(II) .038	-.033	.071
24°	.064	-.056	.120	.039	-.042	.081
20°	.048	-.061	.109	.059	-.056	.115

It appears that, at 32°, 28° and 24°, each limen in the first series is distinctly higher than in the later one at the same temperature. At 20°, however, the discrepancy has largely disappeared, the first series giving a somewhat smaller average value than the second one. On this basis the values from the first series at 32°, 28° and 24° are not used, as being still subject to practise effect; thereafter practise is assumed to have reached and maintained its maximal level.

The limens for 20°, as shown in the graph, are the arithmetic means of the two separate determinations at that level; of the two sets of limens separately determined at 28°, however, (not including the practise set noted in the paragraph above), only one is used in the graph. The reasons for using the one set in preference to the other are two-fold: 1) The set used fits well into the course of the whole curve, while the other departs widely from it. Consider these figures:

	W	C	I
24°	.039	-.042	.081
28° (I)	.046	-.046	.092
28° (II)	.038	-.033	.071
32°	.041	-.038	.079

The slope of the lines from 16° to 24° and from 40° to 32° clearly indicates that each limen at 28° may be expected to be smaller than the corresponding one at 24° and 32°; which

is true of the one set at 28° but not of the other. No change in experimental procedure, or other conditions, to the writer's knowledge, would account for the difference, which seems to be due, in part at least, to the small number of cases in (I). 2) Perhaps more convincing is the relative size of the p.e.'s in the two sets of values:

	p.e.L_w	p.e.$_{L_c}$	p.e.$_I$
28° (I)	.015	.015	.022
23° (II)	.006	.005	.008

It will be seen that the p.e.'s of (I) are about three times as large as those of (II). The evidence is thus all in favor of using (II) rather than (I). If however we get their mean value, weighting them in reverse proportion to the squares of their p.e.'s, these figures result:

(II)	.038 x .015²	-.033 x .015²	.071 x .022²
(I)	.046 x .006²	-.046 x .005²	.092 x .008²
Mean	.039	-.034	.073

It thus appears that if the limens are combined with their approximate weights the resultant mean exceeds by only .001°, in each case, the figure given in the graph. The writer preferred, however, to reject (I) entirely, for the reason that the weighting can not accurately be done. The p.e.'s above used really apply to the crude limens, and not to the 'cor-

rected' ones here used; p.e.'s for the latter are not available. Our weighted mean is, therefore, only an approximation although the great difference in the crude p.e.'s would naturally imply a large difference in the reliability of the corrected limens. Since the weighted mean departs but little from (II) and is, at best, only approximate, the writer saw no reason to use it in the place of (II). He also refrained from weighting the two values of 20° when combining them into a mean. Here the p.e.'s run as follows:

	p.e.$_{L_w}$	p.e.$_{L_c}$	p.e.$_I$
20° (1)	.010	.013	.016
20° (2)	.010	.010	.014

Here the p.e.'s differ by so little that the error from using these approximate weights would presumably, in the writer's opinion, have outweighed a possible increase in the accuracy of the mean. The writer believes, in sum, that the values for both 20° and 28°, as shown in the graph, are the most accurate his data can be made to yield.

The value given for L_w at 44° is unreliable, as shown by an excessive p.e. in one case. The limens for the four time-space orders in the table A II - 44° follow (the p.e.'s given again apply to the corresponding crude limens):

	L_w	p.e.	L_c	p.e.
S-J, L	.099	.055	-.125	.087
S-J, R	.084	.053	-.105	.059
J-S, L	.606	.679	-.061	.058
J-S, R	.051	.040	-.060	.052

All the p.e.'s are here comparable except the one for L_w in J-S, L (.679) - more than twelve times the size of any other in that column. The limen is also of a different order of magnitude from the remaining ones. The value is without question wholly anomalous and may not properly be combined with the others. The mean of the remaining three L_w's (.078) is therefore used as a rough approximation to the true value of L_w at 44°. The very low precision of the Warm integral in J-S, L, affects in some degree the reliability of Xi, whose position is fixed by the intersection of the Warm and Cold curves; the value of the corresponding L_c (-.061), therefore, while quite comparable to the others in that column, is hardly so reliable as they.

The smooth curve fitted to the observed values closely resembles an hyperbola and might have been treated as such; instead, the data were divided into three groups for adjustment: 16°-24°, 24°-32°, 32°-40°. The adjustment is also given for L_c from 32° to 44° and agrees closely with that

from 32° to 40°; but L_w at 44° being a mere approximation cannot be admitted to adjustment. The equations for 16°-24° and 32°-40° are found by the usual least-square method of deriving the most probable rectilinear adjustment to an observed set of values. It will be noted that the linear adjustment in all cases fits the actual figures neatly; the discrepancy never exceeds .001° except in the L_c-adjustment from 32° to 44°, where in one case it equals .002°. The interval 24°-32° not being linear, a quadratic curve with three unknowns is found to fit it; the minimal point of this curve is determined, in the usual way, by equating the first derivative to zero and solving for x. It will be noted that the minima for both L_w and L_c lie within about one degree of 28°.

The Weber ratio is given in each case to the nearest .0005, and is seen to range from .0035 to .0050. This is the lowest fraction ever reported for any sense, but is closely confirmed by observers G and M in the method of limits, 32°-44°; see below, p. 135. As might perhaps be expected, the slope-lines 'round off' at the minimal point instead of meeting sharply at an angle.

The question still remains: To this group of limens, as determined, what degree of reliability may safely be attached? This has been determined by the method shown in the three tables (14 to 16), one for L_w, one for L_c, and one for

TABLE 14

Combined Reliability of the Set of Interval-Differences

	I	PE_I	Diff.	PE_D	Diff./PE_D	Validity of Diff. For	Validity of Diff. Against	Ratio Against/For
44°	Value uncertain							
40°	.156	.0680						
			.039	.0689	..56	.647	.353	.546
36°	.117	.0117						
			.038	.0145	2.63	.962	.038	.040
32°	.079	.0086						
			.008	.0120	.65	.670	.330	.493
28°	.071	.0085						
			.010	.0127	.77	.698	.302	.433
24°	.081	.0094						
			.031	.0141	2.20	.931	. 069	.074
20°	.112	.0106						
			.031	.0204	1.50	.844	.156	.185
16°	.143	.0175						

The product of six ratios = .000064, making less than 7 chances in 100,000 that differences of this size and uniformity of direction would have occurred by chance.

TABLE 15

Combined Reliability of the Set of Successive Differences Between Warm Limens

	L_w	PE_{L_w}	Diff.	PE_D	$\frac{Diff.}{PE_D}$	Validity of Diff. For	Validity of Diff. Against	Ratio Against/For
44°								
40°	.081	.0399						
			.018	.0409	.44	.617	.383	.621
36°	.062	.0086						
			.022	.0108	2.01	.912	.088	.096
32°	.041	.0065						
			.003	.0092	.27	.572	.428	.748
28°	.038	.0065						
			.001	.0089	.09	.524	.476	.908
24°	.039	.0061						
			.015	.0092	1.58	.857	.143	.167
20°	.054	.0069						
			.013	.0136	.99	.748	.252	.337
16°	.067	.0117						

The product of the six ratios = .00228, making 23 chances in 10,000 that differences of this size and uniformity of direction would have resulted by chance.

TABLE 16

Combined Reliability of the Set of Successive Differences Between Cold Limens

	L_c	PE_{L_c}	Diff.	PE_D	Diff./PE_D	Validity of Diff. For	Validity of Diff. Against	Ratio Against/For
44°	.088	.0328						
			.013	.0640	.20	.554	.446	.805
40°	.075	.0549						
			.020	.0556	.37	.599	.401	.669
36°	.055	.0080						
			.017	.0098	1.70	.874	.126	.144
32°	.038	.0057						
			.005	.0079	.68	.677	.323	.477
28°	.033	.0054						
			.009	.0090	.99	.748	.252	.337
24°	.042	.0072						
			.016	.0108	1.55	.852	.148	.174
20°	.058	.0081						
			.018	.0153	1.13	.777	.223	.287
16°	.076	.0130						

The product of the seven ratios = .000622, making less than 7 chances in 10,000 that differences of this size and uniformity of direction would have resulted by chance.

the interval (L_w - L_c). The difference of each limen from the one next to it is shown, together with the p.e. of the difference. We can thus determine, in the usual way, the likelihood, that is, the number of chances in a thousand, that a difference of this size and direction is 'valid' and the likelihood that it is due to 'chance'. We next divide the number of chances opposed to the validity of the difference by the number in its favor. Now, if the size and direction of the successive differences were determined purely by chance, the product of these ratios would approximate unity; hence their actual product is a convenient index of the likelihood that the set of ratios would result from chance. In L_w the product of the six ratios equals .00228, in L_c the product of seven ratios equals.000622, in the intervals the product equals .000064. There are thus a little over two chances in a thousand that the Warm limens would, by chance, have increased regularly on each side of the minimal value at 28° by the amounts here found; in the case of L_c there are something over 6 chances in 10,000; for the intervals, finally, the chances reduce to less than 7 in 100000. Further, these probabilities in favor of chance may be looked upon as liberal, since they are based only upon the size (and direction) of the differences in terms of their p.e.'s; the likelihood that these differences,

in absolute value, should, by pure chance, so closely follow the well-defined uniformity which our values exhibit, is assuredly very much less; but the writer knows no method of reducing this to quantitative terms. In any event, the size of the probabilities, together with the fact that the Weber ratios for the intervals are closely confirmed by two other observers in the method of limits, may serve to show that our results may be accepted as reliable.

A matter of some interest and consequence appears in Table 17, which shows the mean value of Xi at each temperature and, just beneath, the excess of W over C judgments at the same level. (cf. Table 8). It will be recalled that table 8 was compiled for the purpose of determining whether W and C judgments in the various sets of data appeared about equally often, whether the sets were, in that respect, 'symmetrical'. A preponderance of W-judgments, for example, might easily induce a tendency for O to respond with W, and thus artificially lower the L_w. We are now able to approach the question with new light, from the value Xi. The table reveals the curious fact that, in every case but one where W differs from C, the sign of Xi is opposed to that of (W -C). At 16°, for example, the mean value of Xi is -.006°; this means that, apart from time- or space-errors, the comparison jar, when .006° below the standard, is as likely to be judged

TABLE 17

Mean Values of X_i for the Four Time-Space Combinations

28°	24°	20°	16°	20°	24°	28°	44°	40°	36°	32°	28°
-	+	+	-	-	-	+	*	-	-	-	-
.009	.003	.005	.006	.011	.012	.004	.070	.049	.020	.019	.015

Mean Percentage-Frequencies of W and C Judgments: Table 8

W-C	+ .07	- .03	.00	+ .02	+ .03	+ .09	+ .04	- .16	+ .10	+ .08	+ .10	.00

The upper section of this table gives the mean of the four values of Xi (one for each time-space combination) at each temperature-level in Constant method A.

The lower section is taken from Table 8 and shows the percentage of Warmer minus the percentage of Colder judgments at each temperature. For example, at 16°, .38 of all judgments are Warmer and .36 are colder; hence W - C = .02.

Warmer as Cooler; and, when equal to the standard, will be estimated Warmer more often than Cooler. Taken alone, this fact would imply that some tendency is favoring the W-judgment; and the inference seems to be confirmed by the fact that W-judgments were in excess at that level. In 9 cases of 10, where W differs from C, the table exhibits this opposed relation: when W-judgments are in excess, Xi falls below .00°; when C is larger than W, Xi rises above .00°. The only exception to this rule appears in the final 28° of Group I, where both figures are positive. Inasmuch as the limens for this 28° were not admitted to our graph, because of their anomalous character and high p.e.'s, this exception hardly deserves much weight. The two cases where W = C show some departure of Xi from .00° (once +.005°, again -.015°). In these two cases the tendency may be merely a 'hang-over' from the preceding series. In any event, a real inverse correlation seems to exist; in other words, even when W and C differed but little, some degree of expectation or habituation was apparently induced.

The question follows: Will not a tendency of this sort compromise the reliability of our limens? To which may be replied that Xi corrects for the tendency, as well as revealing it. If O's judging is affected with a real W-tendency, the resultant L_w will be too small; but L_c will

be too large, presumably in equal, or nearly the same, degree. Hence, the crude limens will indeed be wrong; but the position of Xi is a clear and convenient index of the degree of displacement. The usefulness of Xi is thus shown in revealing and eliminating a source of error, which the mere combining of the four time-space orders would not touch at all.

We turn now to the time- and space-errors, as given for both crude and corrected limens in Table 18. In accord with Fechner's usage,[1] a time-error is here denominated

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1. Cf. Titchener: Experimental Psychology, Instructor's Manual, Quantitative (1905): p.272.

---

positive when it adds to the apparent temperature of the first stimulus, so that O tends to rate it warmer than he otherwise would; when it serves to subtract from the apparent temperature of the first stimulus, the error is called negative. Likewise, a space-error is positive when it adds to the apparent temperature of the left jar, negative when it subtracts from the apparent temperature of the left jar.

An inspection of Table 18 will show that, in the case of the crude limens, the time-errors (practise series omitted) are uniformly positive at the cooler levels (16° - 28°) and uniformly negative at the warmer levels (44° - 28°). The

TABLE 18

Time- and Space-Errors of Crude Limens

	Practise Series											
	28°	24°	20°	16°	20°	24°	28°	44°	40°	36°	32°	28°
Time L_w	-.041	-.027	.011	.049	.026	.025	.052	-.031	-.085	-.060	-.039	-.001
Error L_c	-.061	-.018	.020	.049	.016	.014	.044	-.072	-.105	-.071	-.030	-.003
Space L_w	-.013	.012	.007	-.011	.001	-.001	-.008	-.020	.016	-.006	.010	.004
Error L_c	-.026	.010	-.004	.002	-.004	.004	-.020	-.005	.025	-.016	.006	-.001

Time- and Space-Errors of Corrected Limens

	Practise Series											
	28°	24°	20°	16°	20°	24°	28°	44°	40°	36°	32°	28°
Time L_w	-.011	-.012	-.007	.001	.007	.008	.001	.014	.002	.006	-.010	.003
Error L_c	-.032	-.004	.002	.001	-.003	-.003	-.006	-.027	-.018	-.006	-.001	.001
Space L_w	-.015	-.004	.010	.002	.003	-.001	.004	-.011	-.002	.007	.003	.003
Error L_c	.024	-.005	-.001	.015	-.001	.003	-.006	.005	.007	-.005	-.002	-.003

principal component of the time-error here is the cooling or warming of the judging surface upon passing through the air from the first stimulus-jar to the second. When the hands are adapted to water below the temperature of the surrounding air, they are slightly warmed upon moving through it, whereas the second jar seems a little cooler than it would otherwise have done. The first jar is thus rated too warm, relatively, and the error is positive. The reverse holds when the air cools the hand. As might be anticipated, the error is greatest at the extreme temperatures - 16° on one side and 40° - 44° on the other; the error does not, however, decrease uniformly from these extremes to the medial temperatures (28° - 32°) for at least two reasons: 1) The air-temperature ranged from about 20° to 35° while these data were being gathered (February 10 - August 11), so that the degree of warming or cooling varied greatly at different stages; 2) The time-error includes other components in addition to the disturbance of adaptation from exposure to the air.

The space-errors show no uniformity of size or direction; in L_w (practise series omitted), 5 are positive and 5 are negative, in L_c, 4 are positive and 6 negative.

In the corrected limens the errors (especially the time-errors) are naturally much reduced. They do not entirely disappear, however, for the reason that the limens in the two opposed time-orders, S-J and J-S, never exactly agree;

comparison of the two therefore always yields some error, positive or negative. Practise series omitted, the time-order J-S has a smaller L_w than does S-J in 8 cases out of 10, and a smaller L_c in 7 cases out of 10.

Constant Method B

These data are shown in the tables under B; for example, B I - 28°.

By this method, the hands upon leaving the adaptation-jar were simultaneously immersed, after a two-second interval, the right hand in the right stimulus-jar and the left hand in the left jar. If the comparison jar (J) was on the left, the judgment, as given, applied to it and is here recorded under L; if the standard was on the left, the judgment applied to the right jar and was tabulated under R. A W under L indicates, therefore, that the left hand, when immersed in a comparison-jar to the left of the standard, felt warmer than did the right hand in the standard; a W under R, likewise, indicates that the right hand, immersed in a comparison-jar to the right of the standard, felt warmer than did the left hand in the standard. The other details of procedure are the same as in method A above.

Let the reader consult Figure 6, giving the graphs and adjustment equations for these data. No values for 44° are available; the range 16° - 40° was covered apart from

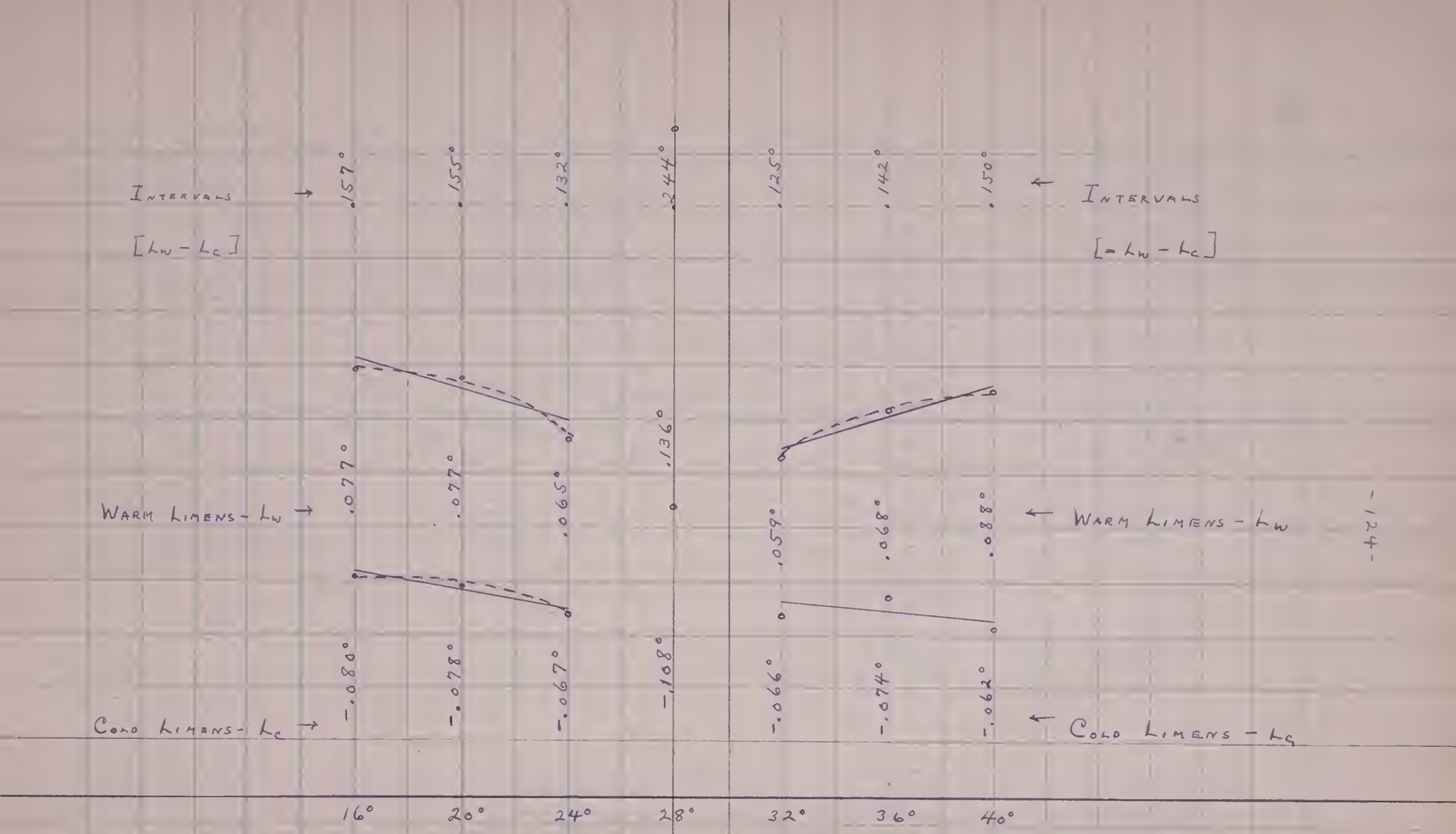


Fig. 6 - Constant Method B.

The values for 28° are omitted from adjustment because entirely anomalous.

Adjustment Equations for Intervals:		Observed and Adjusted Values: 16°	20°	24°	32°	36°	40°
Logarithmic (dotted lines):							
16°-24° $y = .1337° + .0550°1\ (-x)$.	$(x=0$ at $28°)$. Obs.	.157	.155	.132	.124	.142	.150
32°-40° $y = .1253° + .0529°1\ (x)$.	$(x=0$ at $28°)$. Adj.	.160	.150	.134	.125	.141	.151
		$\leq d^2 = .000038$			$\leq d^2 = .000002$		
Linear (continuous lines):							
16°-24° $y = .1604° - .0122°x$.	$(x=0$ at $16°)$. Obs.	.157	.155	.132	.125	.142	.150
32°-40° $y = .1260° + .0128°x$.	$(x=0$ at $32°)$. Adj.	.160	.148	.136	.126	.139	.152
		$\leq d^2 = .000074$			$\leq d^2 = .000014$		

Equations for Warm Limons:		Observed and Adjusted Values:					
Logarithmic (dotted lines):							
16°-24° $y = .0661° + .0268\ 1\ (-x)$	$(x=0$ at $28°)$. Obs.	.077	.077	.065			
32°-40° Omitted because value of 40° seems anomalous. See Text.	Adj.	.079	.074	.066			
		$\leq d^2 = .000014$					
Linear (continuous lines):							
16°-24° $y = .0787° - .0056°\ x$	$(x = 0$ at $16°)$ Obs.	.077	.077	.065	.059	.068	.088
32°-40° $y = .0561° + .0153°\ x$	$(x = 0$ at $32°)$ Adj.	.079	.073	.067	.056	.071	.087
		$\leq d^2 = .000024$					
Equations for Cold Limons:							
Logarithmic (dotted lines):							
16°-24° $y = .0677 + .0283\ 1\ (x)$.	$(x=0$ at $28°)$. Obs.	.080	.078	.067			
32°-40° Omitted because value of 40° seems anomalous. See text.	Adj.	.081	.076	.068			
		$\leq d^2 = .000006$					
Linear (continuous lines)							
16°-24° $y = .0817° - .0066°\ x$	$(x = 0$ at $16°)$. Obs.	.080	.078	.067	.066	.074	.062
32°-40° $y = .0699° - .0025°\ x$	$(x = 0$ at $32°)$. Adj.	.082	.075	.068	.070	.067	.065
		$\leq d^2 = .000014$					

2 An approximate value for the interval at 28° is found from the quadratic adjustment equation for the six intervals (omitting 28°): $y = .1300 - .0019x + .0029x^2$. $(x = 0$ at $28°)$; whence y at 28° equals .130°.
In all equations, each 4°-interval is counted one unit in x.

practise series. The figures for 28° are anomalous, however, and cannot be compared with the others for ~~two~~ three reasons: 1) The figures are much higher than any others in the set, being about twice as large as those for 24° and 32°, adjoining. Consider this table:

	L_w	p.e.	L_c	p.e.	I	p.e.
24°	.065	.016	-.067	.016	.132	.023
28°	.136	.033	-.108	.026	.244	.042
32°	.059	.010	-.066	.011	.125	.014

The course of the adjustment curves would lead us to expect a smaller value at 28°, such as we found in method A, than at either 24° or 32°; instead of that, the actual values are almost twice as large. 2) These figures for 28° average higher than those for the initial 28°, at the very beginning of practise, where L_w was .116, L_c was -.109, the interval being .225. 3) The p.e.'s for the (crude) limens of 28° are from two to three times as large as those for the adjoining values (cf. table above). We thus seem justified in rejecting these values as wholly unreliable; unfortunately, we have no other values to use in their stead.

The relative size of the Warm and Cold limens at 40° is also open to question. The interval (L_w - L_c) agrees well with the values at 32° and 36°; but, whereas at these

points the cold limens exceed the warm, at 40° the cold limen falls far below the warm. Note the table:

	L_w	L_c	I
32°	.059	-.066	.125
36°	.068	-.074	.142
40°	.088	-.062	.150

The reason for this is to be found here: the size of a corrected limen is inversely proportional to its precision, h; that is, $\frac{L_w}{L_c} = \frac{h_c}{h_w}$. The following table gives the values of $h_w + h_c$ (that is, their difference in absolute value, h_c being negative) for the whole of method L. It will be seen that, in row L, practise series being disregarded, the figure at 40° (-.006) is twice as large any other in that row and opposed in sign to any other except the one at 28°, whose values, we saw above, were too unreliable to be used. The fact that h_w, in this one case, is so much smaller in absolute value than h_c, means that the L_w for that temperature is correspondingly enlarged. It will be noted also that, in the other space-order R, h_w and h_c are equal in absolute value at 40°; and that, except for practise series, the R-value is regularly less than the corresponding L-value, but at 40° is greater by .006. We seem justified, therefore,

in looking upon these two limens as open to doubt. As a result, no accurate adjustment to this value was attempted; a linear adjustment is shown for the warm and cold limens from 32° to 40°, but has little significance in each case.

	Practice											
	28	24	20	16	20	24	28	44	40	36	32	28
	-	-							-			-
L	.005	.001	.003	.002	.002	.001	-	-	.006	.002	.002	.001
			-	-	-	-						-
R	.002	.001	.003	.001	.004	.001	-	-	.000	.000	.002	.002

For all other values both the logarithmic and r ectilinear adjustment equations are given; it will be noted that the logarithmic curve gives a neater fit in every case. Upon comparing these results with those from Constant method A, we note several differences of interest: 1) A much lower mean increase in the limens from medial to extreme temperatures, than in A. Compare these figures:

	L_w		L_c		I	
	A	B	A	B	A	B
16°	.067	.077	-.076	-.080	.143	.157
20°	.054	.077	-.058	-.078	.112	.155
24°	.039	.065	-.042	-.067	.081	.132
32°	.041	.059	-.038	-.066	.079	.125
36°	.062	.068	-.055	-.074	.117	.142
40°	.081	.088	-.075	-.062	.156	.150

The limens are widely different at 24° and 32° but of approximately the same size at 16° and 40°. While discrmination,

by method B, is therefore distinctly less accurate in the medial range, it is almost as good at the extremes. 2) A logarithmic increase in B, a rectilinear in A. In other words, by method B, the limens increase at first rapidly then more slowly, as we go toward the extremes.

In explanation of these differences, the writer can offer only a few facts and suggestions. That method B should, in general, give a larger limen might be expected; in method A, both hands are exposed to each stimulus, in method B, only one. As will be seen later, at 32°, with equal practise for each hand, the left hand, alone, gave an interval of uncertainty of .186°, the right hand, alone, one of .394°. If we may assume that the sensitivity of the hands to thermal impressions is inversely proportional to the size of these intervals, then E-O's left hand was 2.1 times as sensitive as his right. When both hands are used together, their discriminative power seems to equal, approximately, that of the more sensitive hand. Thus, practise being equal, the following figures were secured at 32°:

	L_w	L_c	I
Left hand alone	.079	-.107	.186
Both hands	.080	-.100	.180

When the hands are simultaneously immersed in different jars, however, O's discriminative ability obviously depends on both

and may be taken as the mean of the two; in this case, 1.55. It is curious to note that if we multiply the intervals at 32° and 24° (method A) by 1.55, we get .123 and .126, respectively, which agree well with the actual figures in B of .125 and .132. We now come to the second question: Why do the limens in method B increase by a logarithmic rather than a linear curve, as in method A? On this point the writer can merely venture a guess. At levels near skin-temperature, adaptation is relatively quick and easy; when simultaneously immerged, therefore, in jars differing by, say, .10°, the hands tend rapidly to approach indifference, the thermal quality fades out quickly, and discrimination is correspondingly hampered. At extreme temperatures (16° or 40°) adaptation is slow and perhaps never complete; a difference in thermal quality will therefore persist longer and thus facilitate discrimination; hence, the limens would tend to increase more slowly as we depart from skin-temperature. If this explanation is correct, method A may also be affected by this factor, though in smaller degree.

The space-errors for the crude limens are predominantly negative at the cooler temperatures and positive at the higher ones (cf. Table below). A principal factor in the space-error is the greater sensitivity of the left hand, above noted. For example, at 16°, if the left hand were in water

.051° warmer than was the right hand, the two hands would be judged equally cool. At higher temperatures the opposed effect appears - the warmth is more pronounced to the left hand than to the right. In the corrected limens this factor of sensitivity is presumably eliminated; what differences still remain are due to differences in the size of the limens in the opposed space-orders.

Space-Errors for Crude and Corrected Limens (Method B).

		Practise											
		28°	24°	20°	16°	20°	24°	28°	44°	40°	36°	32°	28°
Crude	L_w	+ .017	+ .040	+ .030	− .051	− .068	− .035			+ .091	+ .061	+ .008	− .059
Limens	L_c	− .032	+ .029	+ .011	− .051	− .021	− .015			+ .099	+ .061	+ .012	− .068
Corrected	L_w	+ .012	+ .003	+ .016	+ .006	− .012	− .007			− .018	+ .002	− .002	+ .013
Limens	L_c	− .037	− .008	− .003	+ .006	+ .035	+ .013			− .010	+ .002	+ .002	+ .004

Constant Method C

The data by this method are shown in the tables under C; e.g. C I, 32°.

The method here was in all respects the same as in B above, except that in all cases the hands were adapted to 32° in a special adaptation-jar. The left hand was always immersed in the left stimulus-jar; when the comparison-jar was to the left the judgment was recorded under L; when the standard was on the left, the judgment is shown under R.

The data by this method are extremely fragmentary; in addition to the practise series, only three temperatures were covered. The data are summarized below:

	L_w	pe	L_c	pe	I	pe
32°	.113	.015	-.142	.021	.255	.026
28°	.375	.099	-.387	.103	.762	.143
48°	.133	.029	-.113	.025	.246	.039
36°	.242	.093	-.201	.081	.443	.123
32°	.057	.012	-.078	.019	.135	.023

The size of the limens and of the p.e.'s both clearly show how uncertain discrimination becomes when it proceeds at a temperature widely differing from the one to which the hands are adapted. Perhaps the only point of special interest is the small values at 48°, which is farthest removed from the adaptation-level. These are due to the factor of pain which first clearly emerges at 48° and will be commented on later (see p. 136).

The space-errors are shown below. As in B above, the greater sensitivity of the left hand produces large positive errors at the higher levels.

Space-Errors for Crude and Corrected Limens (Method C).

	32	28	24	20	16	20	24	28	48	44	40	36	32	28
	+	−							+			+	−	
L_w	.032	.139							.140			.250	.059	
	+	−							+			+	−	
L_c	.034	.357							.119			.343	.078	
	−	+							+			−	−	
L_w	.017	.096							.016			.074	.008	
	−	−							−			+	−	
L_c	.015	.122							.005			.019	.027	

Constant Method D

The data are reproduced in the tables under D.

This method was in all respects the same as A above, except that a single hand only was used, and also only the one time-order, S-J.

Data were secured at only one temperature (32°), in the early stages of practise, for the purpose of comparing the sensitivity of the two hands. The figures are:

	L_w	p.e.	L_c	p.e.	I	p.e.
Left hand	.079	.017	-.107	.024	.186	.029
Right hand	.056	.022	-.338	.140	.394	.141

The true limens cannot be determined, since only one time-order was used; but the intervals show that the left hand was 2.1 times as sensitive as the right, if discriminative power is inversely proportional to the size of the intervals of uncertainty. The p.e.'s also reveal the greater pre-

cision of the left hand.

Preliminary Data by the Method of Limits

The following data were secured during one summer-session from four O's,[1] and were preliminary to the main body

1. These observers were: Miss Vitus Mulvey and Miss Bertha Lively, summer-session students, each with a considerable amount of previous work in Psychology; Karl Kraeuter, who had several previous courses in Psychology; and Professor L. R. Geissler, teaching in the summer-session, who was by far the best-trained observer in the group. To all these persons, and in especial degree to Dr. Geissler, the writer wishes here also to acknowledge his indebtedness.

of material above presented. Three forms of the method of Limits were used, distinguished by the letters A, C, and E, which are defined in the notes to the tables.

Five jars were used, the central one being for adaptation to 32°. On its left was a standard, kept constant at the temperature under investigation (32° to 48°); on the left of this standard was the variable jar. On the right of the adaptation jar was another standard, with a variable to its right. The two space-orders could thus be used, variable to left and to right of standard. Both standards were always kept at the same level. The temperature of the variable was raised by means of a large electric heater-lamp, and lowered

by means of a container with melting ice so placed in the jar as not to disturb O's hands.

A descending series began with variable distinctly warmer than standard, passing down through a range of 'Equality' or 'No difference' until the judgment Cooler was given. The ascending series went from distinctly cooler, to warmer. The point midway between the last 'Warmer' and the first 'Equal', in a down-series, was taken as the 'turning-point' or Warm limen for that series; likewise, the point midway between the last 'Equal' and the first 'Cooler' served as the cold limen. Regularly, in one hour, four down and four up series were taken, making eight determinations of each limen. With each mean in the tables is given the number of cases on which it is based.

The procedure in these experiments was not so definitely standardized as in the Constant method, above reported. The time of exposure to each stimulus was approximately constant for each O, but was not subject to exact regulation by a metronome. In methods A and E, only one time-order was used, from standard to variable; the time-error being thus unknown, the true limens cannot be determined; but the 'Equality' intervals (corresponding to the 'Intervals of Uncertainty' ($L_w - L_c$) in our previous results) are available for analysis and comparison.

Notice first the results under Method A, which is analogous to Constant method A. Here both hands were exposed, first to standard, then to variable; adaptation to the same temperature as the standard. The O's, Geissler and Mulvey, covered the range from 32° to 44°. The curve for each one, especially G, is closely linear. The slope of the adjustment lines, that is, the Weber Ratio is .0099 for G, .0085 for M, and .0092 for their mean. These figures strikingly confirm E-O's results by the Constant method, where the slope for the intervals of uncertainty is .0096. It will be noted that these figures apply to the intervals $(L_w - L_c)$; as above stated, the interval cannot be exactly separated into its components, L_w and L_c; but that the slope of their sum, in absolute value, (especially for observer G, the most competent of the group) so closely accords with that found in the Constant method, may be taken as good additional evidence of the reliability of E-O's results. It would not be expected that the actual size of the intervals for E-O would be the same as for G or M, since their absolute sensitivity differs, and the methods are not the same.

We turn now to method E, which differs from method A only in the fact that the hands were adapted to 32° in all cases. All four O's covered the range from 32° to 48° by

this method. The curves are here shown in two groups on the basis of general resemblance. Characteristic of those for G and M is a moderate rise from 36° to 44°, with a pronounced 'jump' at 48°; with K and L, we find a much sharper rise from 32° up to a maximum, followed by a pronounced drop. This difference, which at first looks very curious, may, in the writer's opinion, be explained as follows: At 48° pain becomes prominent; this was repeatedly mentioned by the O's. If it affects the process of judging at all, it may prove either an aid or a hindrance. 1) As a hindrance. If O's temperature-criteria are well in hand, then the emergence of other (secondary) ones may easily be disturbing, at least for a time. Observer G reported, on at least three days of the four at 48°, that the pain served as one, or as the, criterion of judgment. The limen at 48° is thus no longer comparable with the preceding ones at 32°-44°. In the case of G (and also M), the pain-intensities may have been intrinsically harder to discriminate, or harder owing to lack of practise and familiarity with them as criteria. In any event the introduction of the pain quality obviously proved disturbing to the thermal criteria, without supplying others equally accurate. 2) The pain may be an aid to judging, on the contrary, if O's temperature-criteria are less refined or consistent. This

seemed to be especially true in the case of observer L. Her work throughout was erratic; she was extremely conscientious, but never seemed to be quite sure of her criteria. For example, after having given several 'Equals' she would ask E to change them to 'Warmers'; or in a descending series, after having changed from Warmer to Equal, she would revert to Warmer for several judgments. To her the added factor of pain, or stinging, seemed a distinct help. She reported once that it was "easy to tell differences (at 48°); uses pain sensation rather often." Again she reported that on one day she tried to "disregard the stinging sensation and judge only from temperature quality by not keeping hands in water long enough for stinging sensation to develop"; her mean Equality interval for that day was 1.372 as compared with .860 before - an increase of almost 60%. We may conclude, therefore, that, with L (and presumably also with K), the pain criteria were a useful supplement to the thermal ones.

Owing to the pain-factor at 48°, only the values from 32° to 44° were used for adjustment. It will be seen that the logarithmic adjustment fits every one of the four curves, and their means, more closely than does the rectilinear. The curve for L fits both adjustment-curves very badly, as might perhaps be expected from the uncertain character of her judg-

ing; the other three give quite a good approximation to the logarithmic; distinctly better, in each case, than the rectilinear. The question now arises: How explain the logarithmic course of these curves? It must be remembered that in this method the hands are adapted to 32°, while the standard ranges from 32° to 48°; in passing from adaptation to standard, therefore, O encounters an increase of 4° to 16°. We have here a kind of analogy with Fechner's formulation of Weber's law, to the effect that sensation-intensity increases in proportion to the logarithm of the stimulus; if a rise of 4° (32° to 36°) enlarges the limens by a certain amount, we should expect an additional rise of 4° (32° to 40°) to increase them by a somewhat smaller amount, and a third rise of 4°, by a smaller amount still. The rectilinear increase in the limens, found in method A, is thus modified to a logarithmic one by the disturbing effect of the increasing distance between adaptation and standard temperature.

In method C, which is analogous to Constant method C, one hand is exposed to the standard and the other to variable, simultaneously; adaptation to 32° in all cases. The curve for L here again departs widely from that of the other O; the logarithmic adjustment is better for K, and the mean of K and L, while linear and logarithmic are about equally good for L. None of the adjustments fit well here. The logarithmic

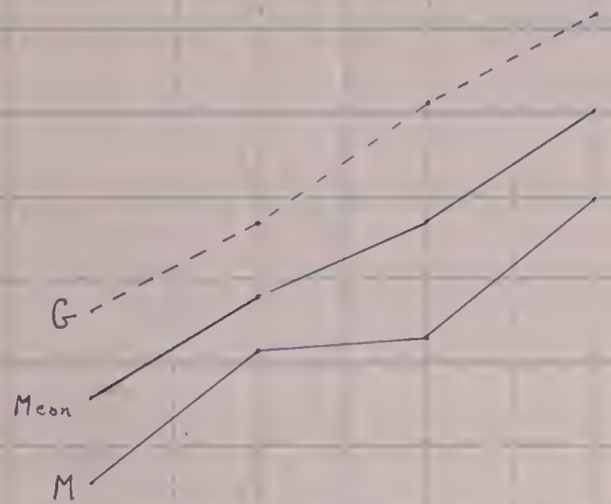
course might be expected here, for the same reasons as in method E above, there being the same increasing distance between adaptation and standard temperature.

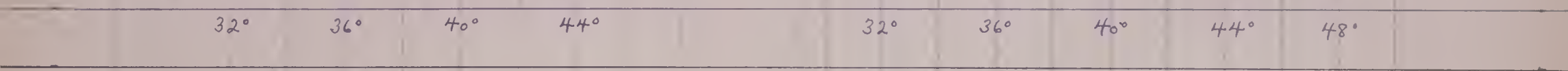
In addition to time- and space-errors, the method of limits usually involves an error of 'expectation'. In a descending series, for example, O knows that he is approaching equality, and may therefore change his judgment from 'Warmer' to 'Equal' sooner than he would have done on the basis of the sense-impression alone. This error of expectation is here tabulated as: (Down-limen minus Up-limen) divided by 2; the error is positive, therefore, whenever the change (from W to E, or E to C) in a down-series appears at a higher point than the corresponding change (E to W, or C to E) in an up-series.

Reference to the tables will show that only one O has a consistent space-error; M uniformly overestimates the variable on the left; this uniformity appears also in her data by method A. The writer does not know how this was caused - perhaps by some difference in the method of immersing the hands. In method C, observer L shows a positive space-error throughout, while K's is predominantly negative. A principal factor here is the difference in sensitivity of the two hands. L's positive error shows that her left hand was more sensitive to warmth than was her right - a fact

confirmed by her introspection. K's right hand, on the contrary, seems more sensitive than his left.

The errors of expectation reveal a striking difference: those of L and K are, in nearly all cases, positive; those of G and M are, almost without exception, negative. L and K, therefore, reveal the influence of expectation, anticipation; while G and M show a perseverative, conservative tendency. Whether, or in what way, these opposed tendencies affect the accuracy of discrimination, cannot be determined from the data. G and M were, in the writer's opinion, the best observers in the group; and the conservative tendency which they reveal may perhaps be taken as the sign of careful observing.





METHOD A: LIMITS

METHOD C: LIMITS.

FIG. 7.

Geissler	.151°	.186°	.234°	.267°	Kraeuter	.115°	.493°	.458°	.533°	.471°
Mulvey	.084°	.135°	1140°	.195°	Lively	.240°	.842°	.941°	L467°	1.165°
Mean	.117°	.160°	.187°	.231°	Mean	.177°	.667°	.669°	1.000°	.818°

Geissler:

$y = .1501° + .0396°x$ ($x = 0$ at 32°)

Mulvey:

$y = .0878° + .0338°x$ ($x = 0$ at 32°)

Mean of Geissler and Mulvey:

$y = .1184° + .0369°x$ ($x = 0$ at 32°).

Agreement of Observed and Adjusted Values

	32°	36	40	44	48	
Obs.	.151	.186	.234	.267		
Adj.	.150	.190	.229	.269		$\Sigma d^2 = .000046$
Obs.	.084	.135	.140	.195		
Adj.	.088	.122	.155	.189		$\Sigma d^2 = .000446$
Obs.	.117	.160	.187	.231		
Adj.	.118	.155	.192	.229		$\Sigma d^2 = .000055$

Coefficient of x is slope per 4°-interval; hence Weber Ratio (=slope per 1°) = .0099 for G, .0085 for M, and .0092 for the Mean.

Kraeuter:

Logarithmic $y = .1701° + .6657° \log x$. ($x=0$ at 28°)

Rectilinear $y = .2169° + .1219°x$. ($x=0$ at 32°)

Obs.	.115	.493	.458	.533	.471
Adj.	.170	.379	.488	.571	$\Sigma d^2 = .020498$
Adj.	.217	.339	.461	.583	$\Sigma d^2 = .036629$

Lively

Logarithmic $y = .2288° + 1.8659° \log x$ ($x=0$ at 28°)

Rectilinear $y = .3055° + .3780°x$. ($x=0$ at 32°)

Obs.	.240	.842	.941	1.467	1.165
Adj.	.229	.790	1.119	1.352	$\Sigma d^2 = .047734$
Adj.	.306	.684	1.062	1.440	$\Sigma d^2 = .044690$

Mean of K and L:

Logarithmic $y = .1989° + 1.2664° \log x$ ($x=0$ at 28°)

Rectilinear $y = .2606° + .2501°x$ ($x=0$ at 32°).

Obs.	.177	.667	.699	1.000	.818
Adj.	.199	.580	.803	.961	$\Sigma d^2 = .020390$
Adj.	.261	.511	.762	1.011	$\Sigma d^2 = .035482$

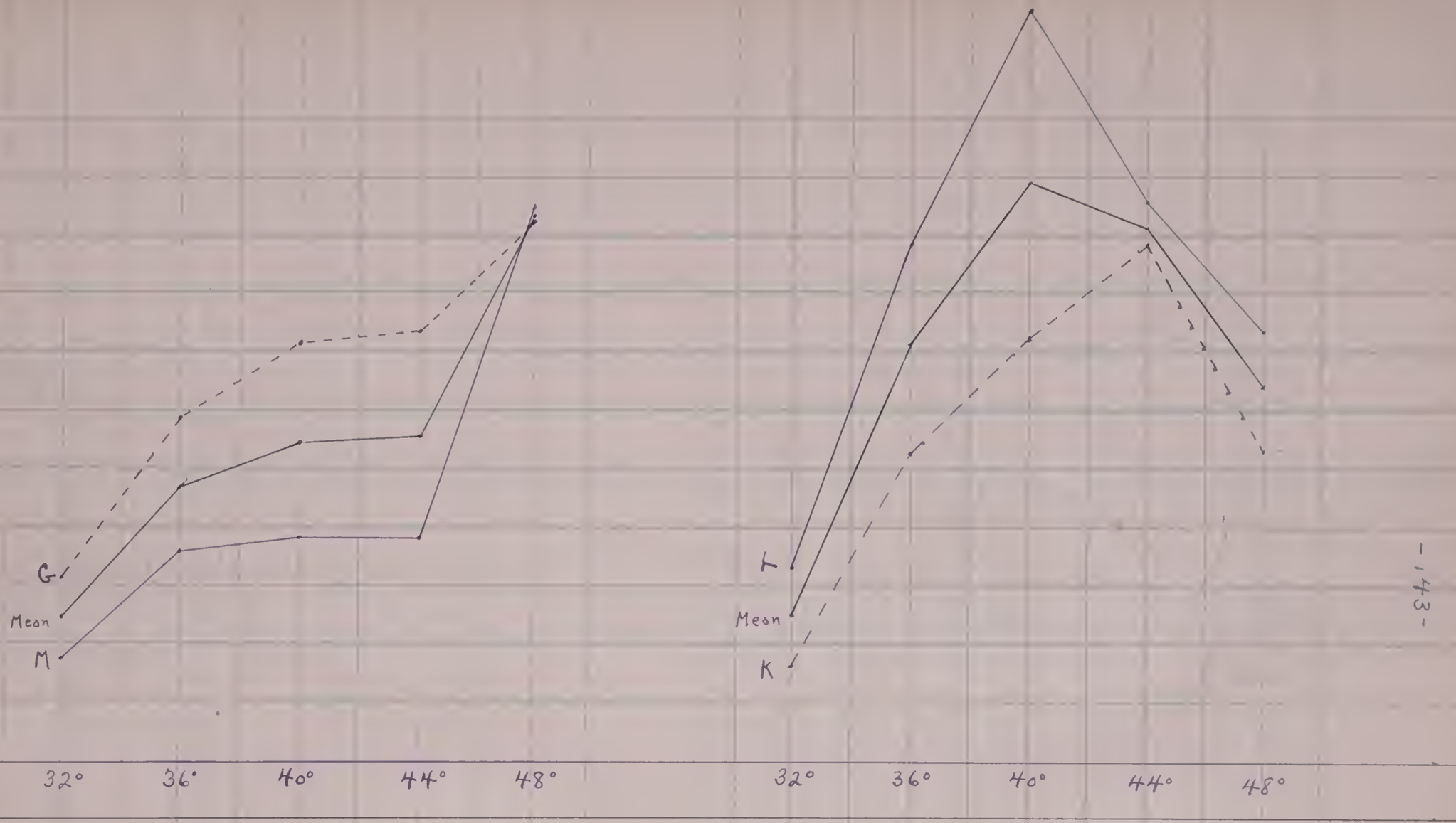


FIG. 8. METHOD E: LIMITS.

Geissler	.151°	.287°	.353°	.360°	.460°	Kraeuter	.078°	.262°	.353°	.437°	.265°
Mulvey	.084°	.176°	.185°	.184°	.472°	Lively	.162°	.438°	.629°	.470°	.367°
Mean	.117°	.231°	.269°	.272°	.466°	Mean	.120°	.349°	.491°	.454°	.316°
General Mean	.119°	.291°	.380°	.363°	.391°						

Method E: Limits

Comparison of Rectilinear and Logarithmic Adjustments:

		Agreement of Observed and adjusted Values:					
		32°	36	40	44	48	
Geissler:	Obs.	.151	.287	.353	.360	.460	
Logarithmic y=.1622°+.3640°log x(x=o at 28°)	Adj.	.162	.272	.336	.381		Σd^2=.001076
Rectilinear y=.1831°+.0696° x (x=o at 32°)	Adj.	.183	.253	.322	.393		Σd^2=.004230
Mulvey:	Obs.	.084	.176	.155	.184	.472	
Logarithmic y=.0982°+.1712°log x(x=o at 28°)	Adj.	.098	.150	.180	.201		Σd^2=.001186
Rectilinear y=.1109°+.0309° x (x=o at 32°)	Adj.	.111	.142	.173	.204		Σd^2=.002429
Mean of Geissler and Mulvey:	Obs.	.117	.231	.249	.272	.466	
Logarithmic y=.1296°+.2685°log x(x=o at 28°)	Adj.	.130	.210	.258	.291		Σd^2=.001092
Rectilinear y=.1468°+.0503° x (x=o at 32°)	Adj.	.147	.197	.248	.298		Σd^2=.003216
Kraeuter:	Obs.	.078	.262	.353	.437	.265	
Logarithmic y=.0791°+.5894°log x(x=o at 28°)	Adj.	.079	.257	.360	.434		Σd^2=.00008
Rectilinear y=.1073°+.1168° x (x=o at 32°)	Adj.	.107	.224	.341	.458		Σd^2=.002870
General Mean:	Obs.	.119	.291	.380	.363	.391	
Logarithmic y=.1370°+.4383°log x(x=o at 28°)	Adj.	.137	.269	.346	.401		Σd^2=.003408
Rectilinear y=.1651°+.0821° x (x=o at 32°)	Adj.	.165	.247	.329	.411		Σd^2=.008957
Lively:	Obs.	.162	.438	.629	.470	.367	
Logarithmic y=.2076°+.6294°log x(x=o at 28°)	Adj.	.208	.397	.508	.587		Σd^2=.032127
Rectilinear y=.2575°+.1115°x (x=o at 32°)	Adj.	.257	.369	.481	.592		Σd^2=.050574

TABLE 19

LIMITS: METHOD A.

	32°		36°		40°		44°		48°	
	L_w	L_c	L_w	L_c	L_w	L_c	L_w	L_c	L_w	L_c
					Space-Errors					
Geissler	-.001	+.001	-.009	-.003	+.029	+.022	+0051	+.071		
Mulvey	+.003	+.004	+.021	+.011	+.021	+.026	+.058	+.037		
					Errors of "Expectation"					
Geissler	-.016	-.013	-.053	-.030	-.103	-.076	-.122	-.098		
Mulvey	-.041	-.051	-.041	-.029	-.053	-.052	-.099	-.103		

LIMITS: METHOD C

					Space-Errors					
Lively	+.001	+.013	+.336	+.387	+.611	+.416	+.366	+.553	+.278	+.455
Kraeuter	+.019	+.025	-.172	-.235	-.348	-.357	-.085	-.077	-.084	+.010
					Errors of "Expectation"					
Lively	+.041	+.025	+.293	+.269	+.396	+.531	+.314	+.544	+.353	+.423
Kraeuter	+.075	+.079	+.098	+.125	+.007	+.134	+.080	+.179	+.163	+.148

TABLE 20

LIMITS: METHOD E

	32°		36°		40°		44°		48°	
	L_w	L_c	L_w	L_c	L_w	L_c	L_w	L_c	L_w	L_c
					Space-Errors					
Geissler	-.001	+.001	-.035	-.032	-.051	-.058	+.017	-.016	+.140	+.144
Mulvey	+.003	+.004	+.038	+.024	+.114	+.092	+.056	+.043	+.060	+.072
Lively	-.004	-.012	-.042	+.017	+.051	+.031	-.028	-.033	+.079	+.144
Kraeuter	+.002	+.008	-.042	-.036	-.038	-.044	+.005	-.001	+.175	+.187
					Errors of "Expectation": 1/2 (↓ Limen − ↑ Limen).					
Geissler	-.016	-.013	+.014	+.011	-.079	-.044	-.074	-.025	-.050	-.068
Mulvey	-.041	-.051	-.022	+.010	-.028	-.003	-.073	-.050	-.087	-.057
Lively	+.003	+.017	+.016	+.049	-.026	+.041	+.023	+.075	-.097	-.095
Kraeuter	+.050	+.044	+.102	+.106	+.040	+.062	+.140	+.038	-.038	+.023

CHAPTER VI

SUMMARY OF CONCLUSIONS

1. With constant stimulus-differences (method A), the two hands being successively exposed to standard and comparison-stimulus, both Warm and Cold limens increase from a minimal value near 28° toward each extreme at 16° and 44°. The values increase with great uniformity and yield the following Weber ratios:

	16° - 24°	32° - 40°
Warm limens (L_w)	.0035	.005
Cold Limens (L_c)	.004	.0045
Intervals (L_w-L_c)	.0075	.0095

The series of values on which these ratios are based has high reliability.

2. In the method of equivalents with constant stimulus-differences, the right and left hands being simultaneously exposed one to standard and one to comparison stimulus, a logarithmic curve is found to fit most closely the values for 16° - 24° and 32° - 40°.

3. With the method of Limits, form A (directly analogous to constant method A in 1. above), the Weber ratio for the mean of two observers is .0092 (for the range 32° to 40°) as compared with .0095 above. The values in 1. are thus closely confirmed by other O's using another method.

4. With the method of limits, adaptation to 32°, logarithmic curves are found to fit most closely the values for the range 32° - 40°. An explanation for this is offered.

5. Pronounced time-errors are found in those methods which involve successive exposure, because the hands are warmed or cooled in passing through the air; space-errors are neither uniform in direction nor large in amount.

6. A marked difference in the sensitivity of the two hands appears in several cases.

7. The error of 'expectation' in the method of limits was of opposite sign in the two groups of observers; the one tending to make an 'expected' change early, the other to make it late.

8. The point Xi, at which the probability of a warm judgment equals that of a cold one, was found very useful in revealing and correcting certain errors not otherwise eliminable.

9. Various departures from customary psychophysical procedure were used and shown to be feasible.

CHAPTER VII

APPENDIX

CONTAINING TABLES OF PRIMARY DATA

Rules by which the Data for the Constant Methods were Tabulated

In making up, from the protocols, the final summaries herein presented, to which the statistical treatment is applied, the writer broached a problem of no little consequence, to whose solution long and careful consideration was devoted. Inasmuch as the temperature of the same jar usually underwent changes of one or more hundredths degree in the course of an afternoon, as revealed by successive readings or checks, the two-fold question arose:

1) What temperature shall be assumed for an hour in which no reading was taken?

2) How widely may the temperature of a jar vary from one reading to the next, and still be accepted as reliable?

The answer to question 1) is fairly obvious, and was uniformly applied in all cases: The temperature for any hour is to be derived by linear interpolation between the readings next before and after.

Example: If a jar reads 36.10° at 1 pm and 36.13° at 4 pm, the temperature at 3 pm, by this rule, equals 36.12°.

Question 2) is far more intricate; in answer to it were formulated, and carefully applied throughout the data, the

following Criteria of Admissible Change in the 'constant' temperatures:

a) If the difference between two successive readings of the same jar does not exceed .10°, the value for each intervening hour, as determined by 1) above, is used with full weight.

Example: If a jar reads 36.05° at 1 pm and 36.09° at 5 pm, the interpolated values at 2 pm (= 36.06°), 3 pm.(=36.07°) and 4 pm (=36.08°) are used with full weight.

b) If the difference between two successive readings of the same jar exceeds .10°, but does not exceed .03° per hour for the time that has elapsed, the value for an hour just before or after either reading is used with full weight; for the second hour before or after is used with half weight.

Example: If a jar reads 36.05° at 1 pm and 36.16° at 5 pm (where the increase of .11° exceeds .10° but amounts to less than .03° per hour for the period of 4 hours) the values, as found by 1) above, for the hours 1-2 and 4-5 are used with full weight, for the hours 2-3 and 3-4 with half weight.

The two formulas above define the basic limits of admissible change, as used in tabulating our data; in addition were developed the following rules, for handling special cases:

c) If only one reading of a given setting is available, that value is used for the hour just before and after with half weight.

Example: If a certain jar is read only once on a given setting, say at 3 pm, that reading is used for the hours 2-3 and 3-4 with half weight; but not at all for any other hours.

d) If the first check-reading of an afternoon agrees with a later one, but not with the scheduled temperature which the jar was to be given that day, then the value for the hour just before the first check is used with half weight.

Example: If a jar, by the original schedule, was to be given the temperature of 36.05°, but, on the first check at 2 pm, reads 36.17°, the discrepancy of .12° exceeds the limits fixed in a) and b) and would normally justify the rejection of the value before this check; but if a later reading at 4 pm confirms the previous one, giving say 36.15°, the former value, 36.17°, is used for the preceding hour, 1-2, with half weight.

e) The value for an hour just before (or after) a reading is used with full weight, when confirmed by the next later (or earlier) check and not opposed by the next earlier (or later)

check; the value for the second hour before or after, under these conditions, is given half weight.

Example: If a jar, on its first reading at 2 pm, equals 36.05° (the original temperature, at the time when the jar was set, not having been recorded) and at 5 pm equals 36.08°, which confirms the earlier reading; then the 2o'clock temperature is used for the hour, 1-2, with full weight, and for the hour, 12-1, with half weight.

f) Values are not extrapolated, but are made equal to the nearest reading, so far as used at all.

Example: If a jar rises from 36.05° at 1 pm to 36.08° at 4 pm, a mean rise of .01° per hour, the value for 5 pm is not extrapolated to 36.09° on this basis, but is taken the same as the figure at 4 o'clock,

g) Values not strictly conforming to these conditions are rejected.

The above formulas had to be derived from general considerations, there being no definite, empirical means of determining just how much discrepancy, or what margin of possible inaccuracy, may properly be tolerated. The writer tried, however, to make his criteria undeniably conservative;

even at the cost, in a few cases, of a good many judgments. Let us consider the rules briefly, in order.

1) As above pointed out (P. 14), the more important changes in the thermostats are gradual rather than abrupt; linear interpolation, therefore, seems to be indicated.

a) Suppose that a jar reads .00° at 12 m. and .10° at 5 pm; the interpolated values for the four intervening hours would then be: .02°, .04°, .06°, .08°. Assume further - what is most unlikely and least favorable to our procedure - that the true temperature continued at .00° until just before the second reading, when it rose abruptly to .10°; the discrepancies between the assumed and the true temperature would then be, in order, as above: .02°, .04°, .06°, .08°; the mean being .05°. It can thus be shown, for any interval, that the mean maximal discrepancy between an interpolated value and the true one, under the least favorable conditions, will not exceed .05°, and in nearly all cases will be very much less.

b) If a jar rises, for five hours, at the mean rate of .03° per hour, it can be shown, as in the paragraph above, that the mean maximal discrepancy, by the least favorable assumption, between an interpolated and the true temperature, will not exceed .075° and will in general be much less. The greater range, however, makes the values less reliable; so

that only those for the hour just before or after a reading are given full weight, while those for an hour second before or after are weighted one-half. This, in the writer's opinion, is a conservative procedure.

c) When only one reading of a given setting was recorded, as happened a few times, there was no means of knowing in what direction or by what rate the thermostat was changing; the values used, therefore, were given only half weight.

d) This rule was devised for a few cases in which the temperature was not set with enough care before work began. If two separate readings, in the course of the afternoon, agreed well with each other but differed widely from the scheduled value, the natural presumption was that the fault lay, not with the thermostat, but in the original setting; the value for the hour preceding the first check was therefore used, though with half weight. It should be noted that, in nearly all cases (cf. p 27), the temperatures were checked either at the time of setting or before any judgments were taken; this rule was needed only in a few cases where this had not been done.

e) In a few cases the original setting was not recorded. If, in these circumstances, two separate readings during the afternoon agreed well with each other, thereby showing that the thermostats were adequately functioning, it was thought

entirely proper to use the value for the hour before the first check at full weight, and for the hour second before with half weight.

f) This rule again applied to few cases but is mentioned for the sake of completeness. It is well recognized that extrapolation, on the basis of a purely empirical curve or course of movement by a variable, is hazardous; there seemed no reason to employ it here, especially since the extrapolated value seldom varied from the one actually used by more than one or two hundredths.

To conclude: the writer needs no admonition that weighting values, whether by zero or one-half, is precarious; but something of the sort was here inevitable. Successive readings of the same jar seldom agree precisely; a thermostat for some reason departs from its normal degree of precision; the absence of a helper deprives the record of some needed checks: the vicissitudes of actual work all combine to give unlike degrees of precision to the data. The writer believes that the above criteria will be accepted as reasonable and conservative; but wishes also, in this connection, to recall and emphasize two facts already given in detail.

1) The high constancy of the thermostats - that is, their small mean change per hour as shown in Table 1; this may serve to prove their general reliability and to justify our

use of some values, in accord with the rules above stated, which had not been checked as completely as desirable. 2) While the number of judgments for temperatures 40° and 44° is undeniably small, owing both to limited time and to the application of our criteria, yet the high general consistency of the data, as shown in Tables 9 and 10, may be taken as evidence that the final results presumably differ but little from the values that a larger number of cases would have yielded.

A few minor details of the tabulating may here be noted:

The temperature of a jar at the close of an hour, whether determined by actual reading or by interpolation, was taken as the temperature for the whole series occupying that hour.

The temperature scheduled for any jar was always an integral multiple of .05°; the tables of data likewise have uniform intervals of .05°. An objective difference ranging from -.02° to +.02° was tabulated as .00°, from .03° to .07° as .05°, and so on. The accuracy of control did not seem to warrant the use of finer intervals.

The judgment 'Equal', which almost never appeared after the practise series, was tabulated as 'Doubtful'.

Notes to the Tables of Data

In the headings of the tables the capital letters A, B, C, D signify the method, as follows:

A: 'The method of constant stimulus-differences; both hands successively exposed to S and J; each constant serving, in turn, as both adaptation-jar and standard.'

B: 'The method of equivalents with constant stimulus-differences; simultaneous exposure of one hand to S and of the other to J; each constant serving, in turn, as both adaptation-jar and standard.'

C: 'Same as B; except that in all cases hands are adapted to 32° in a special adaptation-jar; each constant serving, now as standard, now as comparison stimulus.'

D: 'Same as A, except that a single hand (right or left) was used.'

Of the Roman numerals, I and II, I indicates the first group of temperatures employed, ranging from 32° to 16°; II, the second group, extending from 48° to 28°. (cf. p. 29).

The figures 28°, 24°, and so on, give the temperatures to which the data of the corresponding tables apply. When the same temperature was covered at two different times in the same group, a figure in parenthesis, (1) or (2), shows the order in which it stood.

The words, "Adaptation: 3 min.," mean that an adaptation-period of three minutes preceded each series at that temperature.

The time-order, S-J, means that the standard stimulus (S) preceded the one to be judged (J); J-S reverses this order. L means that the hands, in passing from the first to the second timulus-jar, moved to the left; R, to the right. Thus, in S-J, L, the second jar, J, was to the left of S; in J-S, L, the second jar, S, was to the left of J.

Of the letters, 'W, C, N,' W signifies the percentage of Warmer judgments, C that of Colder, and N the number of cases to which the percentage applies. The percentage of D (Doubtful) judgments is not specially recorded, but in every case equals 100 - (W + C).

In the left margin are given the objective stimulus-differences, in multiples of .05°. If, for example, jar 3 was .05° warmer than jar 1, a judgment applying to 3 was tabulated under +.05°.

At the right side of the page are assembled, in vertical lines, the values derived, by our method of statistical treatment, from the data on the left. h_w is the 'measure of precision' of the 'most probable' Gaussian integral curve that can be fitted to the Warmer-percentages; it equals, as stated above, $\frac{1}{\sigma\sqrt{2}}$; h_c is the same value for the Cooler-

percentages. L_w is the 'Warm' limen; the median, or abscissal point, where the per cent of W equals .50 in the fitted curve; L_c is the 'Cold' limen. The p.e. of L_w and of L_c is derived from the corresponding value of h by the formula: p.e. = $.84535 \frac{1}{h\sqrt{2n}}$ (cf. supra, p. 98). I denotes the 'interval of uncertainty', that is, the distance from the Warm to the Cold limen; $L_w - L_c = I$. The p.e. of this interval is found by the usual formula, applying to two independently measured quantities: $p.e._I = \sqrt{p.e._{L_w}^2 + p.e._{L_c}^2}$. Xi is the 'point of subjective equality' (Urban), the point where the probability of a W-judgment equals the probability of a C-judgment. It is here taken as the 'dividing-point' between Warm and Cold; that is, the point from which the 'true' Warm and Cold Limens are to be reckoned. The 'corrected' value for the Warm limen thus equals L_w - Xi, for the Cold limen equals L_c - Xi. No method of calculating the p.e. of Xi has, to the writer's knowledge, been published; so that its p.e., as well as those for the corrected limens, are omitted.

Preceding the tables we find a Sample Sheet, designed to show how the values just mentioned are computed. The method 'A', space-order 'R', and time-order 'J-S' have been explained earlier in these notes. In the next row the letter 'J' signifies the temperature of the comparison jar (J) with

reference to the standard; for example, at -.05°, J is cooler by .05° than S. The letter 'x' reduces the intervals of .05° to whole units; this facilitates computation and the use of Urban's tables. 'p' is the percentage of W and C judgments; the upper figure applies to W and the lower to C; thus, at -.05°, .09 of all judgments were W and .46 were C. 'N' is the actual number of judgments on which the 'p' is based (38 at -.05°). 'P' is the Urban-Müller weight; thus we see that a 'p' of .09 is weighted .51 in determining the limen while .46 weighs .996. The remaining values: P, xP, x^2P, x P, are all taken from the Urban tables and here multiplied, in each case, by N. The figures for W and C, respectively, are then summed as shown in the rows beginning 'Totals: Warm' and 'Cold'. The terms: h_w, h_c, L_w, L_c, I, Xi, and the 'corrected' limens have been explained in the preceding paragraph. M_w is the mean number of warm judgments per interval and M_c of cold; they are needed for computing the p.e.'s from h, as shown above (p. 99).

SAMPLE COMPUTATION SHEET
Temperature: 32°. Method: A. Space-order: R. Time-Order: J-S.
Date: 2-4.VIII. 19.

J	x	p	N	P	PN	yP	yPN	xP	xPN	x^2P	x^2PN	xyP	xyPN
.15°	3	.75	4	.85	3.40	.40	1.60	2.54	10.16	7.61	30.44	1.21	4.84
		.00		-	-	-	-	-	-	-	-	-	-
.10°	2	.68	22	.92	20.24	.31	6.82	1.85	40.70	3.69	81.18	.61	13.42
		.00		-	-	-	-	-	-	-	-	-	-
.05°	1	.42	24	.985	23.64	.14	3.36	.985	23.64	.985	23.64	.14	3.36
		.08		.47	11.28	.47	11.28	.47	11.28	.47	11.28	.47	11.28
.00°	0	.12	38	.60	22.80	.49	18.62	-	-	-	-	-	-
		.36		.95	36.10	.24	9.12	-	-	-	-	-	-
-.05°	-1	.09	38	.51	19.38	.48	18.24	.51	19.38	.51	19.38	.48	18.24
		.46		.996	37.85	.07	2.66	.996	37.85	.996	37.85	.07	2.66
-.10	-2	.02	28	.19	5.32	.27	7.56	.38	10.64	.75	21.00	.55	15.40
		.84		.69	19.32	.49	13.72	1.38	38.64	2.77	77.56	.97	27.16
Totals:	Warm			4.055	94.78		-39.36		44.48		175.64		48.54
	: Cold			3.106	104.55		- 9.34		-65.21		126.69		-35.78

$$h_w = \frac{(94.78)(48.54)-(-39.36)(44.48)}{(94.78)(175.64)-(44.48)^2} = .432\ (=.022°).$$

$$L_w = \frac{(44.48)(.432)-(-39.36)}{(94.78)(.432)} = 1.431(=.072°).$$

$$h_c = \frac{(104.55)(-35.78)-(-9.34)(-65.21)}{(104.55)(126.69)-(-65.21)^2} = -.483\ (=-.024°)$$

$$L_c = \frac{(-65.21)(-.483)-(-9.34)}{(104.55)(-.483)} = -.808\ (=-.040°).$$

$$N_w = \frac{\Sigma PN}{\Sigma P} = \frac{94.78}{4.055} = 23.37. \quad PE_{L_w} = .8453\ \frac{1}{h_w(2N_w)^{1/2}} = .29\ (=.0145°).$$

$$N_c = \frac{104.55}{3.106} = 33.66. \quad PE_{L_c} = .21(=.0105°).$$

$$PE_I = (.29^2 + .21^2)^{1/2} = .36(=.018°). \quad I = L_w - L_c = 2.239\ (=.112°).$$

$$\chi i = L_c + \frac{h_w I}{h_w - h_c} = -.808 + 1.06 = .252\ (=.013°).$$

Corrected Limens: $L_w = L_w - \chi i = 1.431 - .252 = 1.179(=.059°)$;

$L_c = -1.06\ (= -.053°)$.

-14-

A I - 32° Adaptation: 3 min.

S-J

	L			R		
	W	C	N	W	C	N
.50				1.00	.00	16
.45	1.00	.00	12	1.00	.00	4
.40				1.00	.00	32
.35				1.00	.00	24
.30	1.00	.00	8	1.00	.00	16
.25	1.00	.00	4	1.00	.00	28
.20	1.00	.00	12	.97	.00	32
.15	1.00	.00	16	.98	.00	52
.10				.94	.00	32
.05	.88	.00	16	.63	.00	24
.00	.75	.00	12	.50	.00	12
-.05	.67	.00	24	.13	.13	16
-.10	.31	.06	32			
-.15	.29	.10	52	.06	.31	16
-.20	.00	.50	32	.00	.50	12
-.25	.04	.64	28	.00	1.00	4
-.30	.00	.75	16	.00	.88	8
-.35	.00	1.00	24			
-.40	.00	1.00	32			
-.45	.00	1.00	4	.00	1.00	12
-.50	.00	.94	16			

		h_w	h_c	L_w	PE	L_c	PE	I	PE	X_i	Corrected limens L_w	Corrected limens L_c
S-J	L	.018	-.016	-.083	.015	-.241	.017	.158	.023	-.157	.074	-.084
	R	.021	-.016	.008	.015	-.193	.026	.201	.030	-.077	.085	-.116
	Mean	.020	-.016	-.037		-.117		.180		-.117	.080	-.100

A I - 28° (1) Adaptation: 3 min.

	S-J L W	S-J L C	S-J L N	S-J R W	S-J R C	S-J R N	J-S L W	J-S L C	J-S L N	J-S R W	J-S R C	J-S R N
.35	1.00	.00	4							1.00	.00	4
.30	1.00	..00	8	1.00	.00	12	1.00	.00	6	1.00	.00	4
.25	1.00	.00	4	1.00	.00	12	1.00	.00	10	1.00	.00	6
.20	1.00	.00	20	1.00	.00	8	.75	.00	4	.81	.06	16
.15	1.00	.00	4							.75	.00	4
.10	1.00	.00	12	.88	.00	8	.50	.00	4	.58	.08	12
.05	.55	.10	20	.63	.00	24	.25	.05	20	.40	.00	10
.00	.38	.25	8	.38	.00	8	.25	.17	12	.17	.50	12
-.05	.08	.13	24	.25	.20	20	.00	.60	10	.05	.50	20
-.10	.00	.00	8	.00	.58	12	.00	.92	12	.00	.75	4
-.15				.00	.50	4	.00	1.00	4			
-.20	.00	.38	8	.00	.90	20	.00	1.00	16	.00	1.00	4
-.25	.00	.67	12	.00	1.00	4	.00	1.00	6	.00	1.00	10
-.30	.00	1.00	12	.00	1.00	8	.00	1.00	4	.00	1.00	6
-.35				.00	1.00	4	.00	1.00	4			

	h_w	h_c	L_w	PE	L_c	PE	I	SE	χ_i	Corrected limens L_w	Corrected limens L_c
S-J L	.027	-.009	.038	.014	-.210	.044	.248	.046	-.026	.064	-.184
R	.020	-.023	.016	.019	-.106	.018	.122	.026	-.050	.066	-.056
J-S L	.012	-.037	.121	.039	-.037	.011	.158	.040	.003	.118	-.040
R	.018	-.013	.094	.025	-.035	.032	.129	.041	.039	.055	-074
Mean	.019	-.021	.067	.013	-.097	.015	.164	.020	-.009	.076	-.088

A I - 24° (1) Adaptation: 4 min.

	S-J						J-S					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.35	1.00	.00	4							1.00	.00	2
.30	1.00	.00	8	1.00	.00	8	1.00	.00	8	1.00	.00	10
.25	.95	.00	22	1.00	.00	16	.90	.00	10	1.00	.00	16
.20	.95	.05	22	1.00	.00	8	1.00	.00	2	.72	.06	36
.15	.90	.00	26	.94	.00	16	1.00	.00	6	.58	.00	6
.10	.70	.00	20	.47	.06	17	.64	.04	14	.55	.00	10
.05	.67	.03	18	.67	.06	36	.36	.06	18	.50	.25	2
.00	.25	.50	8	.38	.25	8	.38	.13	24	.13	.42	24
-.05	.13	.45	40	.11	.67	18	.00	1.00	2	.06	.69	18
-.10	.00	.64	18	.13	.56	16	.00	.70	10	.00	.93	14
-.15	.00	.88	16	.07	.52	22	.00	.83	6	.00	1.00	6
-.20	.00	1.00	8	.02	.80	22	.00	.94	36	.00	1.00	2
-.25	.00	.94	16	.00	.95	22	.00	1.00	16	.00	1.00	10
-.30	.00	1.00	8	.00	1.00	8	.00	1.00	10	.00	1.00	8
-.35				.00	1.00	4	.00	1.00	2			

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χi	Corrected limens L_w	Corrected limens L_c
S-J L	.018	-.014	.046	.017	-.052	.023	.098	.029	.004	.042	-.056
R	.015	-.014	.035	.022	-.089	.024	.124	.033	-.025	.060	-.064
J-S L	.011	-.022	.064	.033	-.074	.017	.138	.037	-.028	.092	-.046
R	.015	-.015	.123	.026	+.004	.023	.119	.035	.063	.060	-.059
Mean	.015	-.016	.067	.013	-.053	.011	.120	.017	.003	.064	-.056

A I - 20' (1) Adaptation: 4 min.

	S-J L			S-J R			J-S L			J-S R		
	W	C	N	W	C	N	W	C	N	W	C	N
.55										1.00	.00	4
.40							1.00	.00	2	1.00	.00	2
.30										1.00	.00	2
.25										1.00	.00	2
.20	1.00	.00	8	.92	.00	12	1.00	.00	28	.76	.00	25
.15	.88	.00	16	.83	.08	12	.71	.14	28	.94	.00	31
.10	.81	.03	32	.55	.15	20	.83	.00	26	.74	.00	38
.05	.31	.31	16	.40	.15	20	.63	.00	35	.31	.00	26
.00	.21	.57	28	.29	.25	28	.37	.09	23	.33	.25	24
-.05	.00	.50	20	.13	.56	16	.25	.33	24	.03	.47	32
-.10	.00	.75	20	.03	.75	32	.06	.69	32	.04	.60	26
-.15	.00	1.00	12	.00	.75	16	.09	.64	28	.25	.68	28
-.20	.00	.83	12	.00	1.00	8	.00	.88	16	.04	.79	28
-.25							.00	.75	2			
-.30							.00	.75	2			
-.40							.00	1.00	2	.00	1.00	2
-.55							.00	1.00	4			

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ_i	N Corrected limens L_w	N Corrected limens L_c
S-J L	.027	-.014	.058	.011	-.027	.024	.085	1027	..029	.029	-.056
R	.018	-.015	.070	.019	-.046	.022	.116	.029	.017	.053	-.063
J-S L	.014	-.011	.034	.020	-.078	.032	.112	.037	-.014	.048	-.064
R	.012	-.012	.030	.024	-.074	.024	.124	.034	-.013	.063	-.061
Mean	.018	-.013	.053	.010	-.056	.013	.109	.016	.005	.048	-.061

A I - 16° Adaptation: 5 min.

	S-J L W	S-J L C	S-J L N	S-J R W	S-J R C	S-J R N	J-S L W	J-S L C	J-S L N	J-S R W	J-S R C	J-S R N
.25												
.20	1.00	.00	3	.73	.00	11	1.00	.00	8	.75	.00	8
.15	.36	.18	22	.74	.00	23	.95	.00	20	.86	.00	28
.10	.30	.30	30	.55	.09	11	1.00	.00	12	.60	.10	20
.05	.07	.57	14	.63	.17	48	.84	.00	44	.56	.06	16
.00	.06	.44	18	.41	.18	17	.47	.07	15	.30	.25	20
-.05	.00	.67	48	.21	.43	14	.38	.19	16	.10	.40	40
-.10	.00	.91	11	.17	.47	30	.15	.35	20	.08	.67	12
-.15	.00	.83	23	.09	.77	22	.22	.44	32	.00	.80	20
-.20	.00	.91	11	.00	1.00	3	.00	.38	8	.13	.75	8
-.25												

	h_w	h_c	L_w	PE	L_c	PE	I	PE	Xi	Corrected Limens L_w	Corrected Limens L_c
S-J L	.015	-.011	.179	.020	+.019	.027	.160	.034	.112	.067	-.093
R	.011	-.013	.040	.028	-.085	.022	.125	.036	-.028	.068	-.057
J-S L	.016	-.011	-.034	.019	-.185	.030	.151	.036	-.097	.063	-.088
R	.014	-.015	.059	.024	-.075	.022	.134	.033	-.010	.069	-.065
Mean	.014	-.013	.051	.012	-.082	.013	.143	.017	-.006	.067	-.076

A I - 20° (2) Adaptation: 4 min.

	S-J						J-S					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.45				1.00	.00	2						
.30				.88	.12	4						
.25	.25	.25	4							1.00	.00	4
.20				.25	.33	6	1.00	.00	4	1.00	.00	4
.15	.50	.50	8	.68	.00	28	.92	.00	38	.83	.00	12
.10	.56	.16	32	.65	.05	57	.80	.07	69	.64	.00	39
.05	.46	.21	28	.63	.13	54	.73	.05	84	.66	.08	38
.00	.30	.36	80	.48	.18	80	.55	.16	88	.24	.35	88
-.05	.09	.60	58	.11	.54	28	.15	.60	40	.17	.51	84
-.10	.02	.79	62	.06	.69	32	.28	.50	40	.08	.63	72
-.15	.03	.91	32	.00	.75	8	.25	.33	12	.00	.86	44
-.20	.17	.83	6				.00	.50	4	.25	.50	4
-.25				.00	.75	4	.00	.75	4			
-.30	.00	1.00	4									
-.45	.00	1.00	2									

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χi	Corrected Limens L_w	Corrected Limens L_c
S-J L	.011	-.013	.105	.024	-.016	.020	.121	.031	.042	.063	-.058
R	.010	-.012	.042	.023	-.087	.024	.129	.033	-.027	.069	-.060
J-S L	.015	-.013	-.011	.014	-.111	.019	.100	.023	-.059	.048	-.052
R	.016	-.016	.055	.014	-.054	.012	.109	.019	.000	.055	-.054
Mean	.013	-.013	.048	.010	-.067	.010	.115	.014	-.011	.059	-.056

A I - 24° (2) Adaptation: 4 min.

	S-J						J-S					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.20	1.00	.00	4							1.00	.00	4
.15	.75	.06	16	.92	.00	12	1.00	.00	7	.91	.00	11
.10	.58	.10	52	.80	.00	20	.96	.04	26	.82	.00	49
.05	.31	.34	32	.58	.12	60	.85	.00	53	.61	.12	33
.00	.18	.45	44	.50	.16	44	.64	.00	42	.14	.48	42
-.05	.10	.62	60	.34	.25	32	.48	.30	33	.15	.53	53
-.10	.00	.85	20	.02	.71	52	.16	.49	49	.12	.77	26
-.15	.00	.92	12	.00	.88	16	.00	.73	11	.00	1.00	7
-.20				.00	1.00	4	.00	.75	4			

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ	Corrected Limens L_w	Corrected Limens L_c
S-J L	.018	-.018	.086	.013	-.012	.014	.098	.019	.037	.040	-.049
R	.018	-.021	.017	.013	-.068	.011	.085	.017	-.028	.045	-.040
J-S L	.023	-.016	-.031	.010	-.103	.018	.072	.021	-.061	.030	-.042
R	,021	-.019	.037	.011	-.032	.012	.069	.017	.005	.032	-.037
Mean	.020	-.018	.027	.006	-.054	.007	.081	.009	-.012	.039	-.042

A I - 28° (2) Adaptation: 3 min.

	S-J L W	S-J L C	S-J L N	S-J R W	S-J R C	S-J R N	J-S L W	J-S L C	J-S L N	J-S R W	J-S R C	J-S R N
.30				1.00	.00	4						
.25				.88	.00	8	1.00	.00	3			
.20	1.00	.00	8				.83	.00	6	1.00	.00	4
.15	.50	.00	4	.70	.10	20				1.00	.00	4
.10	.00	.50	8	.50	.00	12	1.00	.00	25	1.00	.00	11
.05	.25	.33	12	.50	.25	4	.86	.00	7	.63	.06	16
.00	.05	.70	20	.35	.30	20	.79	.07	14	.50	.19	18
-.05	.00	.75	4	.00	.67	12	.19	.31	16	.14	.43	7
-.10	.00	.92	12	.80	.50	8	.27	.45	11	.08	.85	26
-.15	.00	.90	20	.00	.75	4	.00	1.00	4			
-.20				.00	1.00	8	.00	1.00	4	.00	.57	7
-.25	.00	1.00	8							.00	1.00	3
-.30	.00	1.00	4									

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ^2	Corrected Limens L_w	Corrected Limens L_c
S-J L	.019	-.012	.132	.025	+.057	.035	.075	.043	.103	.029	-046
R	.011	-.011	.069	.039	-.054	.040	.123	.055	.006	.063	-.060
J-S L	.013	-.022	-.016	.035	-.102	.018	.086	.040	-.070	.054	-.032
R	.022	-.016	.014	.017	-.069	.025	.083	.030	-.022	.036	-.047
Mean	.016	-.015	.050	.015	-.042	.015	.092	.022	.004	.046	-.046

A II - 44°

Adaptation: 5 min.

	S-J L			S-J R			J-S L			J-S R		
	W	C	N	W	C	N	W	C	N	W	C	N
.75							1.00	.00	2			
.70							1.00	.00	2			
.50							1.00	.00	4			
.40							.75	.00	2			
.30				1.00	.00	4	.46	.13	12			
.25	1.00	.00	10	.75	.00	4	.30	.55	10	1.00	.00	12
.20	1.00	.00	8				.00	.75	4	.92	.00	6
.15	.89	.00	19							.56	.13	16
.10	.31	.17	24	.50	.17	4				.63	.09	32
.05	.50	.13	8	.13	.50	8	.08	.50	12	.00	.75	10
.00	.61	.25	14	.11	.39	14	.69	.25	8	.13	.87	8
-.05	.25	.63	8	.00	.88	8	.00	.80	10	.08	.75	12
-.10	.00	.58	6	.06	.71	24	.00	.91	32			
-.15				.11	.58	19	.00	.94	16			
-.20				.00	1.00	8	.00	1.00	6	.00	1.00	4
-.25	.00	.50	4	.00	.85	10	.00	1.00	12	.10	.70	10
-.30	.00	.50	4							.00	.83	12
-.40										.00	1.00	2
-.50										.00	1.00	4
-.70										.00	1.00	2
-.75										.00	1.00	2

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ'	Corrected Limens L_w	Corrected Limens L_c
S-J L	.007	-.006	.057	.055	-.167	.087	.224	.103	-.042	.099	-.125
R	.009	-.007	.177	.053	-.012	.059	.189	.079	.093	.084	-.105
*Obs.	.001	-.007	.807	.067	+.140	.058	.667	.681	.201	.606	-.061
J-S L * Adj.			.279		+.140		.139		.201	.078	-.061
R	.009	-.008	.078	.040	-.033	.052	.111	.065	.027	.051	-.060
Mean	.007	-.007	.148		-.018	.033	.166		.070	.078	-.088

* "Obs." = Observed values; "Adj." = Adjusted values; see text.

A II - 40°

Adaptation: 4 min.

	S-J L W	S-J L C	S-J L N	S-J R W	S-J R C	S-J R N	J-S L W	J-S L C	J-S L N	J-S R W	J-S R C	J-S R N
.50				1.00	.00	4						
.40				1.00	.00	8						
.35							.75	.00	4			
.30				1.00	.00	4						
.25				.83	.00	6	.65	.00	8 1/2			
.20							.37	.26	9 1/2	1.00	.00	4 1/2
.15	1.00	.00	2	.92	.00	6	.61	.09	23	.60	.00	7 1/2
.10	1.00	.00	18	.75	.00	4	.50	.00	8	.57	.03	18 1/2
.05				.58	.08	6	.43	.23	20	.67	.00	3
.00	.50	.00	4	.50	.00	4	.14	.50	25	.36	.44	25
-.05	.83	.00	6				.00	.67	3	.15	.58	20
-.10	.00	.75	4	.17	.36	18	.03	.95	18 1/2	.00	1.00	8
-.15	.25	.17	6	.00	1.00	2	.00	1.00	7 1/2	.04	.70	23
-.20							.00	1.00	4 1/2	.00	.84	9 1/2
-.25	.50	.33	6							,00	1.00	8 1/2
-.30	.25	.50	4									
-.35										.00	1.00	4
-.40	.00	.63	8									
-.50	.00	1.00	4									

	h_w	h_c	I_w	PE	L_c	PE	Int. PE		X	Corrected Limens L_w	Corrected Limens L_c
S-J L	.005	-.003	-.129	.133	-.307	.214	.178	.252	-.196	.067	-.111
R	.010	-.013	.023	.062	-.150	.031	.173	.070	-.075	.098	-.075
J-S L	.007	-.014	.161	.054	+.009	.028	.152	.061	.063	.098	-.054
R	.013	-.012	.073	.030	-.048	.028	.121	.041	.014	.059	-.062
Mean	.009	-.010	..032	.040	-.124	.055	.156	.068	-.049	.081	-.075

A II - 36° Adaptation: 3 min.

	S-J						J-S					
	W	L C	N	W	R C	N	W	L C	N	W	R C	N
.30												
.25							1.00	.00	4			
.20	1.00	.00	8	1.00	.00	15				1.00	.00	4
.15	.75	.00	8	.88	.00	8	.67	.00	3	.67	.00	16 1/2
.10	.94	.00	36	.75	.04	28	.31	.28	32	.71	.04	45
.05	.83	.00	24	.75	.00	12	.23	.24	40	.42	.15	32 1/2
.00	.85	.00	40	.53	.03	40	.05	.69	40 1/2	.26	.21	40 1/2
-.05	.25	.25	12	.25	.21	24	.03	.78	32 1/2	.04	.66	40
-.10	.18	.14	28	.17	.31	36	.00	1.00	41	.06	.88	32
-.15	.00	.75	8	.13	.50	8	.00	.94	16 1/2	.00	1.00	3
-.20	.00	.81	16	.00	.75	8	.00	1.00	4			
-.25										.00	1.00	4
-.30												

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χi	Corrected Limens L_w	Corrected Limens L_c
S-J L	.018	-.023	-.040	.018	-.140	.016	.100	.024	-.097	.057	-.043
R	.015	-.015	.006	.020	-.153	.021	.159	.029	-.074	.080	-.079
J-S L	.019	-.018	.135	.015	+.023	.014	.112	.021	..080	.055	-.057
R	.018	-.026	.068	.014	-.030	.009	.098	.017	.010	.058	-.040
Mean	.017	-.021	.042	.008	-.075	.008	.117	.012	-.020	.062	-.055

A II - 32° Adaptation: 3 min.

	S-J						J-S					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.30												
.25												
.20												
.15	1.00	.00	4							.75	.00	4
.10	1.00	.00	8	1/2	.83	.00 24	.73	.00	28	.68	.00	22
.05	.96	.00	24		.84	.00 32	.42	.16	38	.42	.08	24
.00	.66	.10	58		.62	.19 58	.36	.37	38	.12	.36	38
-.05	.06	.38	32		.13	.42 24	.00	.71	24	.09	.46	38
-.10	.25	.38	24		.00	.70 10	.00	.95	22	.02	.84	28
-.15					.00	1100 4	.00	1.00	4			
-.20												
-.25												
-.30												

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ_i	Corrected limens L_w	Corrected limens L_c
S-J L	.027	-.018	-.026	.009	-.110	.014	.084	.016	-.059	.033	-.051
R	.023	-.025	-.009	.011	-.064	.011	.055	.015	-.038	.029	-.026
J-S L	.016	-.030	.050	.016	-.015	.009	.065	.018	.008	.042	-.023
R	.022	-.024	.072	.014	-.040	.010	.112	.018	.013	.059	-.053
Mean	.022	-.024	.022	.006	-.057	.006	.079	..008	-.019	.041	-.038

A II - 28° Adaptation: 3 min.

	S-J						J-S					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.30												
.25				.88	.00	8						
.20				.83	.00	12	1.00	.00	6			
.15							1.00	.00	9			
.10				1.00	.00	6				1.00	.00	12
.05	.69	.10	44	.65	.09	34	.60	.00	26	.69	.02	48
.00	.39	.30	48	.52	.27	48	.41	.19	48	.29	.28	48
-.05	.22	.47	34	.06	.52	43	1/2 .15	.59	48	.13	.58	26
-.10	.00	.83	6				.00	.92	12			
-.15										.00	.92	12
-.20	.00	.92	12							.00	.88	8
-.25	.00	1.00	8									
-.30												

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ_i	Corrected Limens L_w	Corrected Limens L_c
S-J L	.023	-.020	.015	.010	-.054	.013	.069	.017	-.017	.032	-.037
R	.013	-.024	.030	.020	-.046	.009	.076	.022	-.019	.049	-.027
J-S L	.023	-.040	.024	.010	-.039	.006	.063	.012	-.016	.040	-.023
R	.031	-.023	.025	.007	-.051	.012	.076	.014	-.007	.032	-.044
Mean	.023	-.027	.023	.006	-.048	.005	.071	.008	-.015	.038	-.033

B I

Adaptation: (28°: 3 min.
(24°: 4 min.

	28° (1) L W	28° (1) L C	28° (1) L N	28° (1) R W	28° (1) R C	28° (1) R N	24° (1) L W	24° (1) L C	24° (1) L N	24° (1) R W	24° (1) R C	24° (1) R N
.35							1.00	.00	8			
.30	.86	.00	28	1.00	.00	4	1.00	.00	16	.88	.00	8
.25	1.00	.00	12	.94	.00	16	.92	.00	24	.75	.00	4
.20	.85	.00	48	.75	.00	4	.94	.00	32	.81	.00	16
.15	.85	.00	40	.88	.00	8				.71	.00	24
.10	.64	.00	28	.35	.05	20	.70	.00	20	.50	.08	60
.05	.55	.05	44	.54	.00	24	1.00	.00	8	.31	.13	15
.00	.33	.13	24	.33	.08	24	.25	.13	8	.00	.50	8
-.05	.13	.38	24	.07	.45	44	.38	.19	16	.00	.88	8
-.10	.10	.35	20	.07	.61	28	.20	.36	64	.13	.33	15
-.15	.00	.38	8	.00	.75	40	.21	.32	28			
-.20	.00	.75	4	.04	.69	48	.00	.81	16	.00	.71	28
-.25	.00	.88	16	.00	1.00	12	.00	.50	4	.00	1.00	24
-.30	.00	1.00	4	.00	.86	28	.00	1.00	8	.00	.88	16
-.35										.00	1.00	8

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χ_i	Corrected Limens L_w	Corrected Limens L_c
28° L	.010	-.015	.049	.025	-.127	.023	.176	.034	-.055	.104	-.072
R	.013	-.011	.083	..025	-.190	.022	.273	.033	-.045	.128	-.145
Mean			.066	.018	-.159	.016	.225	.024	-.050	.116	-.109
24° L	.012	-.013	.014	.025	-.164	.024	.178	.035	-.079	.093	-.085
R	.012	-.011	.094	.027	-.106	.031	.200	.041	-.004	.098	-.102
Mean			.054	.018	-.135	.020	.189	.027	-.041	.095	-.094

B I Adaptation: (20°: 4 min.
(16°: 5 min.

	20° (1)						16°					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.35	1.00	.00	4				1.00	.00	2			
.30	1.00	.00	8									
.25							.88	.00	8	1.00	.00	2
.20	.96	.00	24	.50	.00	4	.83	.00	6			
.15	.92	.00	12	.85	.05	44	.53	.10	30	.75	.00	2
.10	.71	.08	24	.42	.08	48	.25	.38	8	.50	.13	16
.05	.57	.13	30	.38	.19	68	.53	.00	15	.69	.00	16
.00	.33	.28	36	.17	.50	36	.00	.43	7	.71	.00	7
-.05	.18	.35	60	.00	.69	36	.13	.67	15	.44	.19	16
-.10	.05	.57	44	.00	.79	28	.07	.47	15	.38	.00	8
-.15	.05	.75	44	.00	.92	12	.00	1.00	2	.13	.60	30
-.20	.25	.50	4	.13	.50	24				.33	.50	6
-.25							.00	1.00	2	.00	.63	8
-.30				.12	.88	8						
-.35				.00	.75	4				.00	1.00	2

		h_w	h_c	L_w	PE	L_c	PE	I	PE	λ	Corrected Limens L_w	Corrected Limens L_c
20°	L	.017	-.014	.038	.016	-.085	.018	.123	.024	-.019	.057	-.066
	R	.008	-.011	.098	.031	-.065	.026	.161	.040	.008	.090	-.071
Mean				.068	.017	-.074	.016	.142	.024	-.005	.073	-.069
16°	L	.011	-.009	.113	.034	-.045	.043	.158	.055	.042	.071	-.987
	R	.008	-.009	.010	.055	-.146	.043	.156	.070	-.072	.082	-.074
Mean				.062	.032	-.095	.030	.157	.044	-.015	.077	-.080

B I Adaptation: 4 min.

	20° (2)						24° (2)					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.20							.75	.00	4			
.15	.91	.00	11	.90	.00	20	.67	.07	15	.90	.00	29
.10	.46	.18	89	.69	.00	59	.73	.05	22	.93	.00	14
.05	.29	.34	108	.69	.05	110	.36	.32	22	.56	.00	18
.00	.31	.33	85	.51	.19	85	.26	.29	34	.46	.11	37
-.05	.21	.53	113	.44	.17	107	.22	.28	18	.36	.36	22
-.10	.15	.39	59	.29	.38	89	.14	.43	14	.23	.42	26
-.15	.05	.55	20	.09	.73	11	.03	.76	29	.07	.73	15
-.20										.00	.75	4

	h_W	h_C	L_W	PE	L_C	PE	I	PE	λ	Corrected Limens L_W	Corrected Limens L_C
20° L	.009	-.007	.122	.019	-.094	.026	.216	.032	.029	.093	-.123
R	.011	-.015	-.014	.015	-.136	.011	.122	.019	-.084	.070	-.052
Mean			.054	.012	-.115	.014	.169	.019	-.028	.082	-.087
24° L	.013	-.012	.072	.026	-.080	.026	.152	.037	.000	.072	-.080
R	.015	-.016	.002	.020	-.110	.020	.112	.028	-.056	.058	-.054
Mean			.037	.016	-.095	.016	.132	.023	-.028	.065	-.067

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B II Adaptation: {40°: 4 min. / 36°: 3 min.

	40° L W	L C	L N		40° R W	R C	R N		36° L W	L C	L N		36° R W	R C	R N
.25					.75	.00	4								
.20	.75	.00	8		.50	.00	4		.50	.50	4		.88	.00	8
.15	.94	.00	16		.40	.28	46		.88	.00	16		.60	.15	36
.10	.58	.05	32		.38	.28	56	1/2	.81	.03	32		.39	.20	48
.05	.54	.04	49	1/2	.14	.43	60		.79	.04	24		.16	.55	44
.00	.51	.13	54		.13	.60	54		.39	.15	70		.28	.26	70
-.05	.50	.11	60		.07	.69	49	1/2	.52	.23	44		.04	.58	24
-.10	.35	.36	56	1/2	.00	.84	32		.17	.42	48		.00	.78	32
-.15	.28	.45	46		.00	1.00	16		.08	.60	36		.00	.94	16
-.20	.00	.75	4		.00	1.00	8		.00	.75	8		.00	1.00	4
-.25	.00	1.00	4												

		h_w	h_c	L_w	PE	L_c	PE	I	PE	X	Corrected Limens L_w	Corrected Limens L_c
40°	L	.007	-.013	-.006	.035	-.164	.017	.158	.039	-.112	.106	-.052
	R	.012	-.012	.176	.021	+.033	.018	.143	.028	.105	.071	-.072
Mean				.085	.020	-.065	.012	.150	.024	-.003	.088	-.062
36°	L	.013	-.011	..004	.019	-.138	.023	.142	.030	-.062	.066	-.076
	R	.012	-.012	.125	.018	-.016	.020	.141	.027	.056	.069	-.072
Mean				.065	.013	-.077	.015	.142	.020	-.003	.068	-.074

B II Adaptation: 3 min.

	32°							28°					
	L			R				L			R		
	W	C	N	W	C	N		W	C	N	W	C	N
.25	.88	.00	8					.83	.00	6			
.20	.95	.00	20					1.00	.00	2	.50	.30	5
.15	.70	.03	20	.82	.00	28		.27	.35	13	.88	.00	4
.10	.67	.05	33	.64	.10	52		.50	.15	13	.90	.00	24
.05	.68	.07	28	.43	.18	84		.37	.31	34	.48	.18	93
.00	.38	.29	56	.26	.18	68		.31	.24	136	.48	.14	134
-.05	.14	.45	83	.00	.62	30	1/2	.26	.39	93	.41	.33	35
-.10	.04	.70	49	.00	.74	33		.02	.73	24	.58	.35	13
-.15	.07	.44	27	.00	.73	20		.13	.75	4	.38	.35	13
-.20				.00	.90	20		.10	.70	5	.25	.75	2
-.25				.00	1.00	8					.00	.67	6

		h_W	h_C	L L_W	PE	L_C	PE	I	PE	χi	Corrected Limens L_W	Corrected Limens L_C
32°	L	.016	-.014	.050	.016	-.080	.016	.130	.022	-.011	.061	-.069
	R	.018	-.016	.065	.011	-.055	.014	.120	.018	.009	.056	-.064
Mean				.058	.010	-.067	.011	.125	.014	-.001	.059	-.066
28°	L	.007	-.008	.130	.033	-.106	.030	.236	.044	.006	.124	-.112
	R.	.004	-.006	.012	.057	-.241	.043	.253	.071	-.137	.149	-.104
Mean				.071	.033	-.173	.026	.244	.042	-.065	.136	-.108

C I Adaptation: 3 min.

	32°						28°					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.35										.75	.00	4
.30				1.00	.00	4	.41	.25	32	.48	.08	26
.25				1.00	.00	12	.33	.25	12	.38	.13	8
.20	1.00	.00	4	.88	.00	8	.35	.35	20	.50	.13	24
.15	.58	.00	12	.54	.00	24	.21	.50	48			
.10	.81	.00	16	.35	.00	20	.25	.42	12	.50	.08	12
.05	.00	.25	4	.11	.11	28	.30	.20	20	.00	.25	4
.00	.17	.08	12	.08	.17	12	.23	.43	40	.33	.33	40
-.05	.18	.11	28	.00	.25	4	.25	.50	4	.40	.35	20
-.10	.05	.25	20	.00	.38	16	.08	.67	12	.50	.33	12
-.15	.08	.29	24	.00	.67	12				.44	.19	48
-.20	..00	.63	8	.00	1.00	4	.08	.71	24	.35	.30	20
-.25	.00	.83	12				.25	.38	8	.25	.25	12
-.30	.00	1.00	4				.00	.71	26	.13	.28	32
-.35							.00	.50	4			

	h_w	h_c	L_w	PE	L_c	PE	I	PE	χi	Corrected Limens L_w	Corrected Limens L_c
32° L	.013	-.013	.072	.027	-.185	.028	.257	.039	-.058	.130	-.127
R	.022	-.014	.135	.015	-.118	.030	.253	.034	.039	.096	-.157
Mean			.104	.015	-.151	.021	.255	.026	-.009	.113	-.142
28° L	.003	-.003	.509	.116	-.036	.113	.545	.162	..230	.279	-.266
R	.002	-.002	.230	.161	-.750	.173	.980	.236	-.241	.471	-.509
Mean			.369	.099	-.393	.103	.762	.143	-.006	.375	-.387

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C II Adaptation: 3 min.

48°

	L W	L C	L N	R W	R C	R N
.30				.75	.00	8
.25	.88	.00	8	1.00	.00	4
.20	.80	.00	15	.44	.03	18
.15	.67	.07	15	.26	.26	21
.10	.66	.10	43	.24	.32	45
.05	.65	.12	34	.12	.54	34
.00	.57	.15	37	.24	.45	37
-.05	.41	.25	34	.09	.63	34
-.10	.39	.17	45	.05	.72	43
-.15	.36	.17	21	.07	.87	15
-.20	.19	.58	18	.07	.80	15
-.25	.00	.50	4	.00	.88	8
-.30	.00	.88	8			

	h_W	h_C	L_W	PE	L_C	PE	I	PE	λ	Corrected Limens L_W	Corrected Limens L_C
48° L	.007	-.007	-.021	.042	-.246	.041	.225	.059	-.138	.117	-.108
R	.007	-.009	.259	.041	-.007	.030	.266	.051	.110	.149	-.117
Mean			.119	.029	-.127	.025	.246	.039	-.014	.133	-.113

C II Adaptation: 3 min.

	36° L			36° R			32° L			32° R		
	W	C	N	W	C	N	W	C	N	W	C	N
.65				.50	.00	2						
.50				.50	.00	2						
.40				.25	.00	2						
.35				.63	.25	4						
.30	.50	.00	4	.13	.13	4						
.25	.75	.00	2	.25	.25	4	.83	.00	9			
.20				.44	.50	8	.81	.00	16			
.15	.40	.20	10	.36	.26	22	.87	.00	15	1.00	.00	5 1/2
.10	.54	.08	12	.00	.94	8	.37	.15	20 1/2	.83	.00	17 1/2
.05	.73	.00	60	.08	.53	30	.22	.33	58	.82	.04	25 1/2
.00	.50	.07	36	.21	.46	36	.27	.30	49	.39	.33	49
-.05	.47	.08	32	.05	.70	60	.04	.82	25 1/2	.39	.29	68
-.10	.56	.00	8	.08	.54	12	.00	.89	17 1/2	.24	.32	20 1/2
-.15	.32	.25	22	.00	.50	10	.00	1.00	5 1/2	.00	.97	15
-.20	.44	.19	8							.06	.75	16
-.25	.50	.00	4	.00	1.00	2				.00	1.00	9
-.30	.63	.13	4	.00	.88	4						
-.35	.38	.25	4									
-.40	.00	.75	2									
-.50	.00	.50	2									
-.65	.00	.50	2									

	h_w	h_c	L_w	PE	L_c	PE	I	PE	K	Corrected Limens L_w	Corrected Limens L_c
36° L	.002	-.003	-.098	.164	-.634	.143	.536	.218	-.414	.316	-.220
R	.005	-.005	.401	.087	.052	.077	.349	.116	.233	.168	-.181
Mean			.152	.093	-.291	.081	.443	.123	-.090	.242	-.201
32° L	.015	-.019	.111	.018	-.005	.012	.116	.022	.046	.065	-.051
R	.015	-.007	-.007	.017	-.160	.037	.153	.041	-.055	.048	-.105
Mean			.052	.012	-.083	.019	.135	.023	-.005	.057	-.078

D I - 32° Adaptation: 3 min.

	Left Hand only						Right Hand only					
	L			R			L			R		
	W	C	N	W	C	N	W	C	N	W	C	N
.50										1.00	.00	2
.35										1.00	.00	2
.30				1.00	.00	4				1.00	.00	4
.25	1.00	.00	12	1.00	.00	8	1.00	.00	14	1.00	.00	10
.20	1.00	.00	12				.95	.05	10			
.15	1.00	.00	12	1.00	.00	12	1.00	.00	12	1.00	.00	14
.10	1.00	.00	16	1.00	.00	12	.94	.00	16	1.00	.00	4
.05	.83	.00	12	.67	.08	12	1.00	.00	8	.83	.00	12
.00	.75	100	12	.75	.00	12	.61	.00	14	.29	.18	14
-.05	.58	.00	12	.42	.00	12	.42	.08	12	.38	.00	8
-.10	.50	.08	12	.13	.06	16	.25	.25	4	.31	.25	16
-.15	.08	.17	12	.00	.50	12	.04	.46	14	.00	.58	14
-.20				.00	.67	12				.10	.30	10
-.25	.00	.38	8	.00	.75	12	.00	.40	10	.00	.64	14
-.30	.00	1.00	4				.00	1.00	4			
-.35							.00	.25	2			
-.50							.00	1.00	2			

	h_w	h_c	L_w	PE	L_c	PE	I	PE	Xi	Corrected Limens L_w	Corrected Limens L_c
Left L	.017	-.013	-.066	.025	-.289	.036	.223	.044	-.161	.095	-.128
Hand R	.019	-.014	-.019	.022	-.168	.031	.149	.038	-.082	.063	-.086
Mean			-.043	.017	-.229	.024	.186	.029	-.122	.079	-.107
Right L	.018	-.003	-.022	.025	-.454	.168	.432	.170	-.085	.063	-.369
Hand R	.012	-.002	-.004	.036	-.359	.223	.355	.226	-.052	.048	-.307
Mean			-.013	.022	-.407	.140	.394	.141	-.069	.056	-.338

Notes to Following Tables of Data.

All the data in the four tables following were secured by some form of the method of limits:

Method A: is analogous to Constant Method A; both hands exposed, first to standard, then to variable; adaptation-jar of same temperature as standard in each case.

Method C: is analogous to Constant Method C; simultaneous exposure of one hand to standard and of the other to variable; adaptation to 32° in all cases.

Method E: both hands exposed first to standard, then to variable; adaptation to 32° in all cases; differs from method A, above, only in adaptation temperature.

With O's Geissler and Hulvey, the adaptation-temperature is given in parenthesis after the temperature of the standard.

'W' signifies the size of the (uncorrected) warm limen, 'C', of the cold limen, and 'I' the interval between them (W-C).

'L', at the beginning of a row, means that the variable (to which the judgment applies) was on the left of the standard; 'R', that it was on the right of the standard.

The symbol '↓' means a descending series, the reverse symbol '↑' an ascending series.

'M' gives the mean of each pair of values directly

above (L and R, Down and Up), their means being the same. With each mean is given the number of cases on which it is based; e.g. N = 32. In general the number of cases is equally divided between L and R; a few times one exceeds the other by several cases.

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Observer: Geissler

Limits: Method A

	32° (32°)			36° (36°)			40° (40°)			44° (44°)		
	W	I	C	W	I	C	W	I	C	W	I	C
L	.044	.153	-.109	.152	.193	-.041	.119	.227	-.108	.057	.287	-.230
R	.042	.149	-.107	.134	.180	-.046	.176	.241	-.065	.159	.248	-.089
	.028	.149	-.121	.090	.164	-.074	.045	.207	-.162	-.014	.243	-.257
	.059	.152	-.094	.196	.209	-.013	.251	.261	-.010	.230	.292	-.062
	N= 48			N=32			N=32			N=32		
M	.043	.151	-.108	.143	.186	-.043	.148	.234	-.086	.108	.267	-.159

Limits: Method E

	36° (32°)			40° (32°)			44° (32°)			48° (32°)		
L	.101	.290	-.189	.328	.347	-.019	-.555	.326	-.881	-1.722	.464	-2.186
R	.031	.285	-.254	.225	.360	-.135	-.581	.393	-.914	-1.442	.456	-1.898
	.080	.290	-.210	.198	.319	-.121	-.612	.311	-.923	-1.632	.479	-2.111
	.052	.285	-.233	.355	.388	-.033	-.464	.408	-.872	-1.532	.442	-1.974
	N=32			N=16			N=32			N=32		
M	.066	.287	-.221	.276	.353	-.077	-.538	.360	-.898	-1.582	.460	-2.042

Observer: Mulvey

Limits: Method A

	32° (32°)			36° (36°)			40° (40°)			44° (44°)		
	W	I	C	W	I	C	W	I	C	W	I	C
L	.058	.085	-.027	.162	.125	+.037	.184	.145	.039	.146	.174	-.028
R	.064	.084	-.020	.205	.146	.059	.226	.135	.091	.236	.216	.047
	.020	.094	-.074	.142	.123	.019	.152	.139	.013	.105	.199	-.094
	.102	.074	+.028	.225	.148	.077	.258	.141	.117	.304	.191	.113
	N=32			N=32			N=32			N=29		
M	.061	.084	-.023	.183	.135	.048	.205	.140	.065	.204	.195	.009

Limits: Method B

	36° (32°)			40° (32°)			44° (32°)			48° (32°)		
L	.181	.161	.020	-.101	.163	-.264	-.407	.171	-.578	-.494	.484	-.978
R	.258	.190	.068	.126	.206	-.080	-.294	.198	-.492	-.374	.460	-.834
	.198	.144	.054	-.015	.160	-.175	-.424	.162	-.586	-.521	.442	-.963
	.242	.208	.034	.040	.209	-.169	-.278	.206	-.484	-.347	.502	-.849
	N=28			N=24			N=32			N=22		
M	.220	.176	.044	.013	.185	-.172	-.351	.184	-.535	-.434	.472	-.906

Observer: Kraeuter

Limits: Method C

	32°			36°			40°			44°			48°		
	W	I	C	W	I	C	W	I	C	W	I	C	W	I	C
L	.061	.121	-.060	.439	.430	+.009	.675	.449	+.226	.286	.542	-.256	.441	.565	-.122
R	.098	.109	-.011	.096	.556	-.460	-.021	.467	-.488	.115	.525	-.410	.274	.377	-.103
	.155	.111	+.044	.366	.467	-.101	.334	.331	+.003	.280	.434	-.154	.521	.486	+.035
	.004	.119	-.115	.170	.520	-.350	.319	.584	-.265	.120	.632	-.512	.194	.455	-.261
	N=32			N=30			N=27			N=32			N=16		
M	.079	.115	-.036	.268	.493	-.225	.327	.458	-131	.200	.533	-.333	.358	.471	-.113

Limits: Method E

	32°			36°			40°			44°			48°		
L	.027	.084	-.057	.014	.267	-.253	-.069	.347	-.416	-1.051	.431	-1.482	-2.442	.276	-2.719
R	.031	.073	-.042	-.070	.256	-.326	-.145	.360	-.505	-1.041	.442	-1.483	-2.091	.254	-2.345
	.079	.084	-.005	.074	.258	-.184	-.067	.331	-.398	-.906	.539	-1.445	-2.305	.204	-2.509
	-.020	.073	-.093	-.131	.264	-.395	-.147	.376	-.523	-1.187	.334	-11521	-2.229	.326	-2.555
	N=32			N=28			N=25			N=26			N=16		
M	.029	.078	-.049	-.028	.262	-.290	-.107	.353	-.460	-1.046	.437	-1.482	-2.267	.265	-2.532

Observer: Lively

Limits: Method C

	32°			36°			40°			44°			48°		
	W	I	C	W	I	C	W	I	C	W	I	C	W	I	C
L.	.061	.252	-.191	.343	.893	-.550	.363	.746	-.383	.142	1.654	-1.512	.359	1.342	- .983
R	.064	.229	-.165	1.015	.791	+.224	1.585	1.136	+.449	.874	1.280	- .406	.915	.988	-.073
	.104	.257	-.153	.972	.866	+.106	1.370	.805	+.565	.822	1.237	-. 415	.990	1.095	-.105
	.021	.224	-.203	.386	.818	-.432	.578	1.076	-.498	.194	1.697	-1.503	.284	1.236	- .952
	N=32			N=28			N=26			N=28			N=14		
M	.062	.240	-.178	.679	.842	-.163	.974	.941	+.033	.508	1.467	- .959	.637	1.165	-.528

Limits: Method E

	32°			36°			40°			44°			48°		
										-.999	.465	-1.464			
L	.046	.153	-.107	.059	.498	-.439	-.160	.609	-.769	-1.055	.476	-1.531	-1.132	.432	-1.564
R	.038	.170	-.132	-.026	.378	-.404	-.058	.649	-.707	-1.004	.418	-1.422	-.984	.302	-1.275
	.045	.147	-.102	.033	.405	-.372	-.135	.561	-.696	-1.050	.522	-1.572	-1.150	.366	-1.515
	.039	.176	-.136	.000	.471	-.471	-.083	.697	-.780				-.956	.368	-1.324
	N=32					N=32	N=24			N=28			N=16		
M	.042	.162	-.119	.016	.438	-.422	-.109	.629	-.738	-1.027	.470	-1.497	-1.053	.367	-1.[illegible]

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